Out-of-plane buckling and design of X-bracing systems with discontinuous diagonals

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Abstract

Several analytical studies have been conducted on the out-of-plane buckling of X-bracing systems. Most of these studies do not consider effects of the mid-span (center) connection and inelastic behaviors of X-bracing system. Usually, one diagonal is interrupted at the center and a gusset plate connects it to the other continuous diagonal and the discontinuous diagonal is modeled as partly pinned. In this study, simple approximated solutions are obtained for the direct evaluation of the elastic buckling loads of X-bracing systems with discontinuous diagonals. The effective length factors are suggested for the general case, i.e., tension and compression diagonals have different section properties, lengths, and axial loads. Inelastic buckling loads of X-bracing systems with discontinuous diagonals are also studied. Inelastic buckling loads are determined using column buckling curves with the suggested effective length factor, and are compared with those from FEM analyses. It is found that inelastic buckling loads of X-bracing systems with discontinuous diagonals from the column buckling curves are over-predicted when the stress level of a tension diagonal is high.

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1. Introduction

An X-bracing system is commonly used in steel structures of industrial and commercial buildings to resist horizontal loads such as wind and seismic loads. An example of X-bracing is shown in Fig. 1. It consists of two slender diagonals, AB and CD, in which diagonal AB is in compression and CD is in tension. In this paper, buckling involving out-of-plane displacement is of interest, so in-plane buckling as shown in Fig. 2(a) is excluded from a possible failure mode. If the diagonals are not connected, the compression diagonal would buckle in a half-sine wave as shown in Fig. 2(b), while the buckled shape would be as shown in Fig. 2(c) when the tension diagonal resists completely the out-of-plane displacement.

For an X-bracing system with a rigid connection at mid-span, extensive studies (i.e., DeWolf and Pelliccione [1]; El-Tayem and Goel [2]; Kitipornchai and Finch [3]; Picard and Beaulieu [4]; Stoman [5,6]; Wang and Boresi [7]) have been conducted. They provide the effective length factor for the X-bracing system with a rigid connection at mid-span. Diagonals can be either simply or rigidly attached at their ends. Segal et al. [8] proposed the closed-form solutions of the effective length factor for an X-bracing system with semi-rigid ends for any value of relative stiffness of the end connections.

All of the foregoing studies are restricted to elastic buckling of the X-bracing system with rigid connection at mid-span. However, the center connection is one of the important factors that can affect the buckling load of X-bracing systems. In practice, one diagonal is interrupted at the center and a gusset plate connects it to the other continuous diagonal. In this case, the discontinuous diagonal is modeled as partly pinned because the gusset plate has a little out-of-plane flexural stiffness compared to the stiffness of the diagonal. These types of X-bracing systems as shown in Fig. 3 are called X-bracing systems with a discontinuous diagonal. Davaran [9] proposed the closed-form solutions of X-bracing systems with
Notations

The following symbols are used in this paper:

- \( a \) Maximum amplitude of deflection;
- \( A \) Maximum amplitude of the initial imperfection;
- \( A_P \) Cross-sectional area of the compression diagonal;
- \( A_T \) Cross-sectional area of the tension diagonal;
- \( e \) Amplitude of the initial imperfection;
- \( E \) Modulus of elasticity of the diagonal member;
- \( E_P \) Modulus of elasticity of the compression diagonal;
- \( E_T \) Modulus of elasticity of the tension diagonal;
- \( f_y \) Yield stress;
- \( f_u \) Ultimate stress;
- \( F \) Maximum frame load;
- \( I \) Moment of inertia of the diagonal member;
- \( I_P \) Moment of inertia of the compression diagonal;
- \( I_T \) Moment of inertia of the tension diagonal;
- \( k \) Effective length factor;
- \( L \) Half length of the diagonal member;
- \( L_P \) Length of the compression diagonal;
- \( L_T \) Length of the tension diagonal;
- \( P \) Axial force in the compression diagonal;
- \( P_{EP} \) Euler buckling load of the compression diagonal;
- \( P_{cr} \) Elastic buckling load of the compression diagonal;
- \( P_{cr} \) Buckling load;
- \( P_{FEM} \) Buckling load from the finite element analysis;
- \( P_y \) Yield load of the compression diagonal;
- \( Q \) Transverse load at the junction between the tension and the compression diagonal;
- \( r \) Radius of gyration of the compression diagonal;
- \( S \) Lateral stiffness;
- \( T \) Axial force in the tension diagonal;
- \( T_{FEM} \) Axial force in the tension diagonal from the finite element analysis when the system reaches buckling;
- \( T_y \) Yield load of the tension diagonal;
- \( \lambda \) Buckling parameter.

a discontinuous diagonal. However, the solution proposed by Davaran [9] is only applicable to X-bracing systems with identical diagonals (the tension and compression diagonals have same length, material and section properties) and its solution cannot give the effective length factor of X-bracing systems with discontinuous diagonals directly. In order to prevent the out-of-plane buckling of X-bracing systems, it is crucial to fully understand the inelastic out-of-plane buckling behavior as well as the out-of-plane elastic buckling behavior. However, research on the inelastic behaviors of X-bracing systems (DeWolf and Pelliccione [1]) is scarce in the literature.

In this study, simple approximated solutions are obtained for the direct evaluation of the elastic buckling loads of X-bracing systems with discontinuous diagonals. The elastic buckling loads and the effective length factor are suggested for the general case, i.e., the tension and compression diagonals have different section properties, lengths, and axial loads. Inelastic buckling behaviors of X-bracing systems are also studied. Using the effective length factor suggested for the elastic buckling, inelastic buckling loads are determined from the Korean Standard and AISC column buckling curve equations. The predicted inelastic buckling loads are compared with
2. Elastic buckling of X-bracing systems

In this section, elastic buckling loads and effective buckling length factors of X-bracing systems with discontinuous diagonals are investigated. An X-bracing system with the discontinuous tension diagonal is considered first followed by the system with the discontinuous compression diagonal. The results of this study are compared with those of previous researchers and finite element analyses.

2.1. X-bracing systems with a discontinuous tension diagonal

The X-bracing system with the discontinuous tension diagonal is shown in Fig. 4 where $T$ is the axial force in the tension diagonal; $P$ is the axial force in the compression diagonal; $L_T$ is the length of the tension diagonal; and $L_P$ is the length of the compression diagonal. Fig. 5(a) and (b) show analytical models of the discontinuous tension diagonal and the compression diagonal, respectively. In this paper, it is assumed that the boundary conditions at both ends of the discontinuous tension diagonal are pinned since the size and flexural stiffness of a gusset plate are very small when compared to the tension diagonal. The resulting equilibrium equation for the tension diagonal, assuming an elastic behavior, is

$$y'' - \beta^2 y = -\frac{Qx}{2E_T I_T}; \quad \beta = \sqrt{\frac{T}{E_T I_T}};$$

$$0 \leq x \leq \frac{L_T}{2} \quad (1)$$

where, $E_T$ is the modulus of elasticity of the tension diagonal; $I_T$ is the moment of inertia of the tension diagonal; and $Q = \delta/2$ where $\delta$ is the transverse load at the junction between the tension and the compression diagonal. The deformed shape of the discontinuous tension diagonal is described by $y = ax$. The assumed deformed shape of the discontinuous tension diagonal satisfies the boundary conditions of the discontinuous tension diagonal $[y(0) = y''(0) = y''(L_T/2) = 0]$, where $a$ is the maximum amplitude of deflection. Substituting $y = ax$ into Eq. (1) and using the Galerkin method, the mid-span displacement can be expressed as

$$\delta = \frac{L_T^4}{4T} Q. \quad (2)$$

The lateral stiffness $S$ furnished by the discontinuous tension diagonal is then

$$S = \frac{4T}{L_T}. \quad (3)$$

This represents the stiffness of an equivalent bracing at the center of the compression diagonal. Timoshenko and Gere \cite{10} show that the relation between the elastic buckling load $P_{cr}^E$ and $S$ and the resulting value of $P_{cr}^E$ is

$$P_{cr}^E = P_{EP} + \frac{3SL_P}{16} \leq 4P_{EP}; \quad P_{EP} = \frac{\pi^2 E_P I_P}{L_P^2} \quad (4)$$

where, $S$ is the lateral stiffness; $L_P$ is the length of the compression diagonal; $E_P$ is the modulus of elasticity of the compression diagonal; $I_P$ is the moment of inertia of the compression diagonal; and $P_{EP}$ is Euler buckling load of the compression diagonal.

The effective length factor $k$ is defined as

$$k = \sqrt{\frac{P_{EP}}{P_{cr}^E}} = \sqrt{\frac{P_{EP}}{P}}. \quad (5)$$

and using Eqs. (3)–(5), the effective length factor $k$ can be found as

$$k = \sqrt{1 - 0.75 \frac{L_P T}{L_T P}} > 0.5. \quad (6)$$

Eq. (6) represents the effective length factor $k$ of the X-bracing systems with a discontinuous tension diagonal. The
effective length factor is independent of flexural stiffness of the tension and compression diagonals because of the assumptions made in the derivation, Eq. (6). The assumptions are: The joints are pinned at the both ends of the discontinuous tension diagonal. The deformed shape of the discontinuous tension diagonal is described as $y = ax$.

The effective buckling length factor is the function of $T/P$ and $L_P/L_T$. If $k = 0.5$, the discontinuous tension diagonal resists completely the out-of-plane displacement and then the compression diagonal would buckle in the second-mode. Using Eq. (6), the second-mode buckling occurs when

$$\frac{T}{P} \geq \frac{L_T}{L_P}$$

(7)

For example, if the lengths of the tension and compression diagonals are identical ($L_T = L_P$), the second-mode buckling occurs when $T/P \geq 1$.

Davaran [9] proposed the closed-form solution of X-bracing systems with a discontinuous tension diagonal in order to evaluate the elastic buckling load $P_{cr}^E$, and gave

$$T/P = \frac{1}{1 - \tanh \nu L/E}; \quad v = \sqrt{\frac{P_{cr}^E L^2}{E I}}$$

(8)

where, $L$ = the half length of the diagonal member; $E$ = the modulus of elasticity of the diagonal member; and $I$ = the moment of inertia of the diagonal member. From Eq. (8), $P_{cr}^E$ is determined and then the effective length factor can be evaluated using Eq. (5). Eq. (8) cannot evaluate the effective length factor directly and Eq. (8) is only applicable to X-bracing with identical diagonals. The results of this study are compared with those of Davaran [9] and finite element analysis. Buckling analysis was performed using the structural analysis program ABAQUS [11]. 3-node beam elements (B32), using quadratic functions for displacement interpolation, and a box-shape cross-section were used. The tension and the compression diagonals have the same section properties ($E_T I_T = E_P I_P$), and lengths ($L_T = L_P = 5\, \text{m}$). Fig. 6 shows the effective length factor $k$ for X-bracing systems with a discontinuous tension diagonal from Eqs. (6) and (8), and finite element analysis. The $x$ and $y$ axes represent the ratio of the tension and the compression $T/P$ and the effective length factor $k$, respectively. With $T/P$ increasing, $k$ values keep decreasing from 1 to 0.5. Eq. (6) provided excellent agreement with other results.

2.2. X-bracing systems with a discontinuous compression diagonal

Fig. 7 shows X-bracing systems with a discontinuous compression diagonal. The analytical model of the tension diagonal and the discontinuous compression diagonal are shown in Fig. 8(a) and (b), respectively. The equilibrium equation for the tension diagonal in Fig. 8(a) is equal to Eq. (1). The deformed shape of the tension diagonal shown in Fig. 8(a) is assumed as

$$y = a \sin \frac{\pi x}{L_T}$$

(9)

where, $a$ = the maximum amplitude of deflection. Unlike the assumed shape for the discontinuous tension member, sine function is used for the assumed displacement to guarantee the
continuity of the slope at the center. The assumed deformed shape of the continuous tension diagonal satisfies the boundary conditions \( y(0) = y(L_T) = y''(0) = y''(L_T) = 0 \). Using Galerkin’s method with the weight function same as the deformed shape (Eq. (9)), the equilibrium equation for the tension diagonal is simplified as

\[
\frac{\pi^2}{4L_T} + \frac{T}{4EI_T} L_T^2 Q = \frac{2\pi^3 E_T I_T}{2\pi^2 E_T I_T}.
\]

Using Eqs. (9) and (10), \( Q \) can be expressed in terms of the mid-span displacement \( \delta[y(0.5L_T)] \) of the tension diagonal as

\[
Q = \frac{\pi^2(E_T I_T/L_T^2 + T)\delta}{2L_T}.
\]

The lateral stiffness \( S \) furnished by the tension diagonal is then

\[
S = \frac{\pi^2(E_T I_T/L_T^2 + T)}{2L_T}.
\]

The equilibrium equation of the discontinuous compression diagonal shown in Fig. 8(b) is given by

\[
y'' + \alpha^2 y' = -\frac{S0}{2P}; \quad \alpha = \sqrt{\frac{P}{E_P I_P}};
\]

\[
0 \leq x \leq \frac{L_p}{2}
\]

by applying the boundary conditions, including \( y''(0) = y''(L_p/2) = 0 \), and \( y(0) = 0 \), the preceding equation results in

\[
\left( \frac{L_p}{2} - \frac{2E_P I_P \alpha^2}{S} \right) \sin \left( \alpha \frac{L_p}{2} \right) = 0
\]

whose solutions are

\[
P_{cr}^E = \frac{S L_p}{4},
\]

\[
P_{cr}^E = \frac{4\alpha^2 E_P I_P}{L_p^2}.
\]

Eq. (15a) is the elastic buckling load of the discontinuous compression diagonal and Eq. (15b) is the upper-bound solution of the Eq. (14). Using Eqs. (12), (15a) and (15b), the effective length factor as defined in Eq. (5) becomes

\[
k = \sqrt{\frac{1 - 1.23 \frac{L_p}{2} \frac{f_y}{E_P}}{1.23 \frac{L_p}{2} \frac{f_y}{E_P} \frac{L_T}{L_T} + \frac{P_{cr}^E}{P_{cr}^E}}} \geq 0.5.
\]

Eq. (16) represents the effective length factor of X-bracing systems with a discontinuous compression diagonal. The effective length factor of X-bracing systems with a discontinuous compression diagonal is never less than 0.5 from the upper-bound solution where the second-mode occurs. Then, the compression diagonal would buckle in the second-mode when

\[
\frac{T}{P} \geq 0.82 \frac{L_T}{L_p} - 0.25 \frac{E_T I_T L_p^2}{E_P I_P L_T^2}.
\]

Fig. 9. Comparison of \( k \) (X-bracing system with discontinuous compression diagonal).

Fig. 9 shows a criterion for the second-mode buckling of X-bracing systems with a discontinuous compression diagonal. Davaran [9] derived the closed-form solution of X-bracing systems with the discontinuous compression diagonal to evaluate the elastic buckling load \( P_{cr}^E \), and gave

\[
\tan \left( \sqrt{\frac{T}{P}} (\nu L) + \sqrt{\frac{T}{P}} (\nu L) (\frac{T}{P} - 1) \right) = 0;
\]

\[
\nu = \sqrt{\frac{P_{cr}^E L_p^2}{EI}}.
\]

Again, it is found that Eq. (18) is too complex to solve directly for \( P_{cr}^E \) and only applicable to identical diagonals. The effective length factor \( k \) of the X-bracing system with a discontinuous compression diagonal proposed in this study Eq. (16) was compared with results obtained from other methods. Fig. 9 shows the comparison of \( k \) values based on Eqs. (16) and (18), and finite element analyses. From the Fig. 9, it can be observed that the discontinuous compression diagonal buckled in the second-mode when \( T/P > 0.57 \). The same result can be obtained from Eq. (17). Results of this study provided good predictions of the elastic buckling load of X-bracing systems with a discontinuous compression diagonal.

3. Inelastic buckling of X-bracing systems

3.1. Buckling load from column bucking curves

The effect of inelasticity on the buckling load of the X-bracing system depends on its material and geometric properties. To determine inelastic buckling load \( P_{cr} \), the buckling parameter \( \lambda \) is defined first as

\[
\lambda = \sqrt{\frac{P_{cr}}{P_{cr}^E}} = \frac{kL_p}{r \pi} \sqrt{\frac{f_y}{E_P}}.
\]

where, \( P_y \) or \( f_y A_P \) = the yield load of the compression diagonal; \( f_y \) = the yield stress; \( A_P \) = the cross-sectional area of the compression diagonal; \( r \) = the radius of gyration of
the compression diagonal; and $P_{cr}^E$ or $P_{cr}^c = \pi^2 E I_P/(kL_P)^2$ is the elastic buckling load of the compression diagonal. In Eq. (19), the effective length factor $k$ is assumed to be Eq. (6) for X-bracing systems with a discontinuous tension diagonal and Eq. (16) for X-bracing systems with a discontinuous compression diagonal. The buckling parameter $\lambda$ is then calculated using Eq. (19). Once $\lambda$ is determined, the buckling load $P_{cr}$ can be evaluated using a column buckling curve. In this study, two kinds of column buckling curves are used. One is the Korean Standard [12] and the other is the AISC [13] column buckling curve. The equation of the Korean Standard [12] column buckling curve is

$$\frac{P_{cr}}{P_y} = 1 \quad \text{for } \lambda \leq 0.2;$$

$$\frac{P_{cr}}{P_y} = 1.109 - 0.545\lambda \quad \text{for } 0.2 < \lambda \leq 1.0;$$

$$\frac{P_{cr}}{P_y} = \left(\frac{1}{0.773 + \lambda^2}\right) \quad \text{for } \lambda > 1.0$$  \hfill (20)

and the equation of the AISC [13] column buckling curve is

$$\frac{P_{cr}}{P_y} = 0.658\lambda^2 \quad \text{for } \lambda \leq 1.5;$$

$$\frac{P_{cr}}{P_y} = \left(\frac{0.877}{\lambda^2}\right) \quad \text{for } \lambda > 1.5.$$  \hfill (21)

3.2. Buckling load from FEM

The inelastic buckling loads of X-bracing systems from the column buckling curves were compared with finite element analyses using ABAQUS [11]. To verify the finite element modeling approach used in this study, simulation results of inelastic buckling load were compared with experimental results from DeWolf and Pelliccione [1]. The X-bracing systems were tested in the frame shown in Fig. 1. The members of the frame were made up of pairs of 76 mm × 130 mm channels. The cross-sectional properties of members are given in Table 1 and the first dimension in Table 1 is the in-plane dimension. The buckling loads from the finite element simulation and the maximum frame loads $F$ from the test are compared in Table 2. The simulation results agree closely with the test results. It is concluded that the finite element model provides a good estimate of the inelastic buckling load.

The inelastic behavior of X-bracing systems from a finite element analysis was determined by taking into account the effect of large deformations, material inelasticity, initial imperfection, and residual stresses. The members of the X-bracing systems for this study had the cross-section of the box-shape. The height was 50–200 mm; the width was 100–400 mm; and the thickness was 5–10 mm. The length of the tension and the compression diagonals used in the analysis was 5 or 10 m. Ratios of $T/P$ were varied from 0 to 1. The X-bracing systems were assumed to have a set of residual stresses shown in Fig. 10 which satisfies the bending and axial force equilibrium conditions. The distribution of the residual stress used was based on experimental results [14]. The tri-linear elastic–plastic stress–strain relationship shown in Fig. 11 was used. The yield stress was $f_y = 250$ MPa; the modulus of elasticity was $E = E_p = E_T = 210000$ MPa; and the ultimate stress was $f_u = 400$ MPa. Fig. 12(a) and (b) show assumed shapes and amplitudes of the initial imperfection adopted in this study. A combination of the first two modes of the compression diagonal is used to represent the interactive buckling mode. Equations of the initial imperfections are given by

$$e = A \left[\sin(\pi x/L_P) + \sin(2\pi x/L_P)\right]$$  \hfill (22a)

$$e = A \left[x + \sin(2\pi x/L_P)\right] \quad \text{for } 0 \leq x \leq L_P/2$$

$$e = A \left[1 - 1/L_P (x - L_P) + \sin(2\pi x/L_P)\right]$$  \hfill (22b)

for $L_P/2 \leq x \leq L_P$

where, $e$ = the amplitude of the initial imperfection; and $A$ = the maximum amplitude of the initial imperfection. Eqs. (22a) and (22b) represent the assumed shape of initial imperfection of the continuous compression diagonal and the discontinuous compression diagonal, respectively. The values of $e_{max}$ used are 0, $L_P/1500$, and $L_P/1000$. The values of $L_P/1500$ and $L_P/1000$ are the maximum allowable magnitudes of an initial imperfection given by the Korean Standard code [12] and AISC [13], respectively.
and (a) and (b) show the variations in non-dimensional load and the plastic limit, which suggests that the results residual stress showed a good agreement with the Euler buckling parameter, \( \lambda \), which was not selected smoothly as a function of initial imperfections. This caused reduction of the lateral stiffness \( S \) of tension diagonals. Even though \( T \) was smaller than \( T_y \), the tension diagonal partially yielded because of residual stresses and geometrical imperfections. This caused reduction of the lateral stiffness \( S \) of the tension diagonal, hence, reduction of the buckling load of the X-bracing system occurred. Magnitude of the reduction of the lateral stiffness \( S \) furnished by the tension diagonal is dependent on the cross-section dimension and forces acting on the tension diagonal. Also, the buckling parameter, \( \lambda \), is determined from combination of parameters listed in Eqs. (6), (16) and (19). It should be noted that a set of parameters which produce a value of \( \lambda \) is not unique. Each set of parameters for \( \lambda \) was selected so that \( \lambda \) could be adequately spaced (14 data points). \( \lambda \) is dependent on \( T/P \) which was not selected smoothly as a function of \( \lambda \). Therefore, when the lateral stiffness was reduced due to excessive tension.

3.3. Inelastic buckling load comparisons

Inelastic buckling loads of the X-bracing systems with a discontinuous diagonal were calculated from the column buckling curves using the procedure described in Section 3.1 and compared with the non-linear finite element analyses using the model verified in Section 3.2.

**Fig. 12.** Assumed shapes and amplitudes of initial imperfections: (a) For the continuous compression diagonal; (b) For the discontinuous compression diagonal.

### Table 1
Dimensions and properties of test members

<table>
<thead>
<tr>
<th>Test number</th>
<th>Size of compression diagonal (mm)</th>
<th>Size of tension diagonal (mm)</th>
<th>Yield stress of compression diagonal (MPa)</th>
<th>Yield stress of tension diagonal (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25.4 × 22.23</td>
<td>25.4 × 22.23</td>
<td>224.61</td>
<td>224.61</td>
</tr>
<tr>
<td>2</td>
<td>25.4 × 22.23</td>
<td>25.4 × 11.11</td>
<td>224.61</td>
<td>254.24</td>
</tr>
<tr>
<td>3</td>
<td>25.4 × 19.05</td>
<td>25.4 × 19.05</td>
<td>276.98</td>
<td>276.98</td>
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<td>4</td>
<td>25.4 × 19.05</td>
<td>25.4 × 9.53</td>
<td>231.50</td>
<td>322.45</td>
</tr>
<tr>
<td>5</td>
<td>25.4 × 15.88</td>
<td>25.4 × 15.88</td>
<td>268.02</td>
<td>268.02</td>
</tr>
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<td>6</td>
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<td>268.02</td>
<td>209.46</td>
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<td>7</td>
<td>25.4 × 12.70</td>
<td>25.4 × 6.35</td>
<td>248.04</td>
<td>302.47</td>
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</table>

### Table 2
Maximum frame loads from tests and finite element analyses.

<table>
<thead>
<tr>
<th>Test number</th>
<th>Maximum frame load, Test (kN)</th>
<th>Maximum frame load, FEM (kN)</th>
<th>Errors (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>164.12</td>
<td>163.76</td>
<td>−0.22</td>
</tr>
<tr>
<td>2</td>
<td>104.75</td>
<td>97.01</td>
<td>−7.98</td>
</tr>
<tr>
<td>3</td>
<td>131.10</td>
<td>129.50</td>
<td>−1.24</td>
</tr>
<tr>
<td>4</td>
<td>73.68</td>
<td>72.98</td>
<td>−0.96</td>
</tr>
<tr>
<td>5</td>
<td>104.13</td>
<td>97.90</td>
<td>−6.36</td>
</tr>
<tr>
<td>6</td>
<td>80.81</td>
<td>75.21</td>
<td>−7.46</td>
</tr>
<tr>
<td>7</td>
<td>50.91</td>
<td>48.95</td>
<td>−4.00</td>
</tr>
</tbody>
</table>

from buckling load with the effective length factor of X-bracing systems having a discontinuous tension diagonal can predict the FEM results well. This is consistent with the assumption that the initial imperfections and residual stress are not considered in obtaining the buckling load. When initial imperfection and residual stress were introduced, the non-dimensional compression \( P_{FEM}/P_y \) was reduced from 10% to 30%. Lower buckling loads were predicted with \( e_{max} \) value of \( L_p/1500 \) than with \( L_p/1000 \) because of the larger magnitude of initial imperfections. The maximum reduction of \( P_{FEM}/P_y \) due to initial imperfections was 7%, while the average reduction was 3.8%. It was found that \( P_{FEM}/P_y \) with residual stress and initial imperfection was not smooth and some analysis results deviated from the buckling curve in the inelastic region when \( T_{FEM}/T_y > 0.6 \). This phenomenon occurred because of the reduction of lateral stiffness \( S \) of tension diagonals. Even though \( T \) was smaller than \( T_y \), the tension diagonal partially yielded because of residual stresses and geometrical imperfections. This caused reduction of the lateral stiffness \( S \) of the tension diagonal, hence, reduction of the buckling load of the X-bracing system occurred.

**Table 2**

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<td>131.10</td>
<td>129.50</td>
<td>−1.24</td>
</tr>
<tr>
<td>4</td>
<td>73.68</td>
<td>72.98</td>
<td>−0.96</td>
</tr>
<tr>
<td>5</td>
<td>104.13</td>
<td>97.90</td>
<td>−6.36</td>
</tr>
<tr>
<td>6</td>
<td>80.81</td>
<td>75.21</td>
<td>−7.46</td>
</tr>
<tr>
<td>7</td>
<td>50.91</td>
<td>48.95</td>
<td>−4.00</td>
</tr>
</tbody>
</table>
force in tension diagonal with the residual stress, the buckling load does not vary continuously as a function of \( \lambda \) as shown in Fig. 13(a). This can be also confirmed by Fig. 13(b) where, \( T_{\text{FEM}}/T_y \) is not continuous as a function of \( \lambda \). One more interesting point is that when the tension and compression diagonal yielded simultaneously (i.e. finite element result with \( \lambda = 0.32 \) in Fig. 13(a)), the buckling load did not decrease even though \( T_{\text{FEM}}/T_y > 0.6 \).

Fig. 14(a) and (b) represent \( P_{\text{cr}}/P_y \) or \( P_{\text{FEM}}/P_y \) with \( \lambda \), and variations in \( T_{\text{FEM}}/T_y \) with \( \lambda \) for the X-bracing systems with the discontinuous compression diagonal, respectively. Similar observations with Figs. 13(a) and (b) can be found. Some of the results of \( P_{\text{FEM}}/P_y \) with the effects of residual stress and initial imperfection show disagreement with the buckling curve. Again, due to the residual stress and geometrical imperfection, the lateral stiffness \( S \) of the tension diagonal is reduced. This leads to reduction of buckling load even when \( T_{\text{FEM}} < T_y \). It is concluded that the buckling load of an X-bracing system is sensitive to residual stress distribution and the stress level of the tension diagonal.

Motivated by the results presented in Figs. 13 and 14, the non-smooth distribution of the FEM results, a set of additional finite analysis are performed for X-bracing systems with a discontinuous diagonal to investigate the effect of the stress level of the tension diagonal on the buckling load (Figs. 15 and 16). Based on the geometry of the cross-section and residual stress distribution of the X-bracing system in the study, the column buckling curves provided higher buckling loads compared with the finite element results when the normalized stress level of the tension diagonal \( T_{\text{FEM}}/T_y \) is larger than 0.6 as shown in Fig. 15. The finite element results provided higher buckling loads when \( T_{\text{FEM}}/T_y < 0.6 \) as shown in Fig. 16. This result suggests that the column buckling curve with the suggested effective length factor might over-predict the inelastic buckling load since the calculation does not incorporate the effect of imperfection and residual stress in the X-bracing system especially when the stress level in the tension diagonal is relatively close to the yield stress. However, it is noted that when the tension and compression diagonal yielded simultaneously, the buckling load from the finite element analysis did not decrease even though the stress level in the tension diagonal is relatively close to the yield stress.

It should be also noted that the analysis results are specific to the cross-section and the residual stress distribution used in this study. However, with further thorough parametric studies as...
well as experiments, it is expected that the proposed method can be used as an out-of-plane design criterion for the X-bracing system with a discontinuous diagonal.

4. Conclusions

In this study, an approximated solution for the effective length factors which are used for determining the out-of-plane buckling load of X-bracing systems with a discontinuous diagonal (Eqs. (6) and (16)) was suggested. The factors were obtained for a general case where the tension and the compression diagonal have different section properties, lengths, and axial loads. If the tension diagonal has a sufficient flexural stiffness, the tension diagonal resists completely the out-of-plane displacement. Then the compression diagonal would buckle in the second-mode. The second-mode buckling occurs when Eqs. (7) and (17) are satisfied for X-bracing systems with a discontinuous tension and compression diagonal, respectively. Especially for identical diagonals, the second-mode buckling occurs when $T/P \geq 1$ for the X-bracing system with a discontinuous tension diagonal and $T/P \geq 0.57$ for X-bracing system with a discontinuous compression diagonal.

Inelastic out-of-plane buckling loads of X-bracing systems with discontinuous diagonals were also studied. Inelastic buckling loads were calculated from the suggested effective length factor for X-bracing systems with a discontinuous diagonal and the column buckling curves from the Korean Standard and AISC. Results were compared with those from non-linear finite element analyses taking into account the effects of large deformations, material inelasticity, initial imperfections, and residual stresses. The column buckling design curves over-predicted the inelastic out-of-plane buckling load of the X-bracing system when the stress level of the tension diagonal was relatively high. It is expected that, with further calibration of the design equations, the proposed method can be used to determine the accurate out-of-buckling load for designing the X-bracing systems with a discontinuous diagonal.

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References