Shear strength and design of trapezoidally corrugated steel webs

Jiho Moon, Jongwon Yi, Byung H. Choi, Hak-Eun Lee

Abstract

Due to the accordion effect, corrugated steel webs are only able to resist shear force. The shear force in the web can cause three different buckling modes: local, global and interactive shear buckling. Although several researchers have been investigating it, the shear buckling behavior of the corrugated webs has not yet been clearly explained, this leads to conservative design. This paper presents the shear strength and design of trapezoidally corrugated steel webs. Firstly, global shear buckling equations are rearranged in order to derive the global shear buckling coefficient. The interactive shear buckling coefficient and the shear buckling parameter for corrugated steel webs are then proposed based on the 1st order interactive buckling equation. The inelastic buckling strength is determined from the buckling curves based on the proposed shear buckling parameter. A series of tests are conducted to verify the proposed design equations. From the test results of this study and those provided by previous researchers, it was found that the proposed shear strengths provide good predictions for the shear strength of the corrugated steel webs.

1. Introduction

Trapezoidally corrugated steel webs, as shown in Fig. 1(a), provide enhanced shear buckling strength and weight savings due to elimination of the need for transverse stiffeners. While PSC box girder bridges with corrugated steel webs have been extensively constructed in France and Japan, the first PSC box girder bridge constructed with corrugated steel webs has recently been constructed in South Korea [1]. In addition, applications of corrugated steel webs have been extended to extra-dosed and cable-stayed bridges [2], and, there have been several attempts to apply corrugated webs to plate girder bridges [3–5].

For a corrugated steel web, it is assumed that the web carries only shear forces due to the accordion effect [6]. Because of this characteristic, the corrugated steel webs fail due to shear buckling or yielding. Three different shear buckling modes (local; global; and interactive) are possible, depending on the geometric characteristics of corrugated steel webs. Fig. 1(b) shows the geometric notations of the corrugated steel webs used in this study. In Fig. 1(b), a is the flat panel width; b is the horizontal projection of the inclined panel width; c is the inclined panel width; d is the corrugation depth; t_w is the web thickness; and θ is the corrugation angle.

Several researchers have conducted extensive studies on the shear buckling of corrugated webs [1,2,7–10]. Easley and McFarland [7] proposed the global shear buckling equation of corrugated webs by treating the corrugated web as an orthotropic flat web. Elgaaly et al. [8] and Yamazaki [9] conducted experimental studies on the buckling characteristics and strength of corrugated webs. Recently, Yi et al. [1] studied the nature of the interactive shear buckling of corrugated webs, and concluded that the 1st order interactive shear buckling equation that does not consider material inelasticity and material yielding provides a good estimation of the shear strength of corrugated steel webs. Based on test results and finite element analyses, Driver et al. [10] suggested shear design criteria for corrugated webs that are suitable for bridge design specification. However, despite the significant amount of research that has been carried out, the shear buckling designs of corrugated webs have been conducted with a large margin of safety.

This paper presents shear strength and design criteria of trapezoidally corrugated webs, based on the 1st order interactive equation proposed by Yi et al. [1], and test results for the shear buckling of corrugated webs. Global and interactive shear buckling equations are rearranged into a form of the classical plate buckling equation. The global and interactive shear buckling coefficients $k_G$...
Notations

The following symbols are used in this paper:

- flat panel width;
- horizontal projection of the inclined panel width;
- width of flange;
- inclined panel width;
- bending stiffness per unit length about x axis (strong axis);
- bending stiffness per unit length about y axis (weak axis);
- corrugation depth;
- average corrugation depth;
- maximum corrugation depth;
- minimum corrugation depth;
- Young’s modulus of elasticity;
- yield stress;
- shear modulus of the flat plate;
- shear modulus of the corrugated plate;
- web height;
- global shear buckling coefficient;
- interactive shear buckling coefficient;
- local shear buckling coefficient;
- web thickness;
- thickness of flange;
- length of girder;
- Applied load;
- maximum fold width;
- ratio of flat panel width to inclined panel width (w/c);
- global buckling factor that depends upon the boundary condition;
- vertical displacement;
- factor which is introduced to provide a margin of safety on global buckling strength;
- shear strain;
- length reduction factor defined as (a+b)/(a+c);
- local buckling slenderness;
- shear buckling parameter;
- corrugation angle;
- critical shear buckling stress;
- shear buckling strength obtained by finite element analysis;
- elastic local shear buckling stress;
- elastic interactive shear buckling stress;
- elastic global shear buckling stress;
- shear yielding stress;
- Poisson’s ratio.

and k_l are then derived as functions of d/t_w and w/h_w, where, w is the maximum fold width, and h_w is the web height. The shear buckling parameter of corrugated webs, λ_s, is then proposed. The shear buckling strength, considering material inelasticity, residual stresses, and initial imperfections, can be determined from the buckling curves using the proposed λ_s. A series of tests are performed with large corrugated webs in order to verify the proposed shear strength. Proposed shear strengths are also compared with those suggested by previous researchers [10, 11]. From the results, the proposed shear strengths were successfully verified and provided a good estimation of the shear strength of the corrugated webs.

2. Elastic shear buckling of trapezoidally corrugated steel webs

2.1. Local shear buckling

The presence of local shear buckling is characterized by the buckling of individual sub-panels. It is assumed that corrugated webs are treated as a series of flat rectangular sub-panels supporting each other along their vertical edges and by flange along their horizontal edges. The elastic local shear buckling stress of the corrugated webs, τ_{cr,L}^c, can be determined by the classical plate buckling theory [12] and expressed as

\[ τ_{cr,L}^c = k_l \frac{π^2 E}{12(1−υ^2)} \left( \frac{w}{h_w} \right)^2 \]  

(1)

where E is Young’s modulus of elasticity; υ is Poisson’s ratio; w is the maximum fold width (maximum of flat panel width a and inclined panel width c); and t_w is the web thickness. k_l is the local shear buckling coefficient. Assuming that the panel has simply supported edges, k_l is given by

\[ k_l = 5.34 + 4 \left( \frac{w}{h_w} \right)^2 \]  

(2)

k_l is a function of the aspect ratio of the sub-panel, w/h_w. Table 1 represents profiles of existing bridges in France and Japan that have corrugated webs. It is found that the w/h_w on actual bridges that have been constructed to date, are generally smaller than 0.2, as shown in Table 1. Fig. 2 shows the variations in k_l with w/h_w. The difference between Eq. (2) and k_l = 5.34 is smaller than 2.9% when w/h_w ≤ 0.2. Therefore, k_l = 5.34 is recommended for practical design purposes.

Table 1

<table>
<thead>
<tr>
<th>Name of bridge</th>
<th>a (mm)</th>
<th>b (mm)</th>
<th>d (mm)</th>
<th>c (mm)</th>
<th>η</th>
<th>w/h_w</th>
<th>d/t_w</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sinkai bridge</td>
<td>250</td>
<td>200</td>
<td>150</td>
<td>250</td>
<td>0.90</td>
<td>0.21</td>
<td>16.67</td>
</tr>
<tr>
<td>Matunoki bridge</td>
<td>300</td>
<td>260</td>
<td>150</td>
<td>300</td>
<td>0.93</td>
<td>0.14</td>
<td>15</td>
</tr>
<tr>
<td>Hondani bridge</td>
<td>330</td>
<td>270</td>
<td>200</td>
<td>330</td>
<td>0.91</td>
<td>0.10</td>
<td>22.22</td>
</tr>
<tr>
<td>Cognac bridge</td>
<td>353</td>
<td>319</td>
<td>150</td>
<td>353</td>
<td>0.95</td>
<td>0.20</td>
<td>18.75</td>
</tr>
<tr>
<td>Maupre bridge</td>
<td>284</td>
<td>241</td>
<td>150</td>
<td>284</td>
<td>0.92</td>
<td>0.11</td>
<td>18.75</td>
</tr>
<tr>
<td>Dole bridge</td>
<td>430</td>
<td>370</td>
<td>220</td>
<td>430</td>
<td>0.93</td>
<td>0.17</td>
<td>22</td>
</tr>
</tbody>
</table>

Fig. 1. I-girder with corrugated steel webs: (a) Profiles of I-girder with corrugated webs; (b) Geometric notations.
2.2. Global shear buckling

Global shear buckling is characterized by the formation of diagonal buckles through the entire web similarly to a flat plate web. Easley and McFarland [7] proposed the global shear buckling stress of the corrugated webs by treating the corrugated web as an orthotropic flat web. Elastic global shear buckling stress of the corrugated steel webs \( \tau_{cr,c} \) is given by

\[
\tau_{cr,c} = \frac{36\beta}{\pi^2 h_w^2} \left[ \frac{D_y^{1/4} D_x^{3/4}}{w} \right] \eta
\]

where \( \beta \) is the global buckling factor that depends on the boundary condition. \( \beta \) is equal to 1 and to 1.9 for simply supported and fixed edges, respectively. Conservatively, for practical design purposes, \( \beta = 1 \) is generally used. \( D_y \) and \( D_x \) are the bending stiffnesses about the x and y axes, respectively, as shown in Fig. 1. \( D_y \) and \( D_x \) are defined as

\[
D_y = \frac{E t_w^3}{12 (1 - \nu^2) \eta}
\]

\[
D_x = \frac{E t_w^2 \left( \frac{d}{t_w} \right)^2 + 1}{6 \eta}
\]

where \( d \) is the corrugation depth; and \( \eta \) is the length reduction factor defined as \( (a + b)/(a + c) \). Substituting Eq. (4) into Eq. (3), the elastic global shear buckling stress of the corrugated webs, \( \tau_{cr,c} \), can be expressed as

\[
\tau_{cr,c} = \frac{\pi^2 E t_w^2}{12 \left( 1 - \nu^2 \right) \eta} \left( \frac{t_w}{h_w} \right)^2
\]

In Eq. (5), the global shear buckling coefficient, \( \kappa_G \), is defined as

\[
\kappa_G = \frac{36\beta}{\pi^2 h_w^2} \left[ \frac{D_y^{1/4} D_x^{3/4}}{w} \right] \eta
\]

In Eq. (6), generally, \( \beta = 1; \nu = 0.3 \) (for steel); and the length reduction factor \( \eta \) varies in the range of 0.9 to 1 for actual bridges that have been constructed to date, as shown in Table 1. Variations in \( \kappa_G \) with \( d/t_w \) are shown in Fig. 3. The values of \( \kappa_G \), when \( \eta = 1 \) are smaller than those for \( \eta = 0.9 \) and the difference is 5.1%. Therefore, for a conservative design with negligible errors, \( \eta = 1 \) can be used without consideration of the corrugation profiles. Substituting \( \beta = 1; \nu = 0.3 \); and \( \eta = 1 \) into Eq. (6), and considering \( d/t_w \geq 10 \), \( \kappa_G \) is simplified as

\[
\kappa_G = 5.72 \left( \frac{d}{t_w} \right)^{1.5}
\]

Eq. (9) always gives a lower value than local and global shear buckling strength. Therefore, elastic shear buckling stress can easily be determined using Eq. (9) without the need to calculate the elastic local and global shear buckling stress.

For a practical design, \( k_l \) is simply calculated with \( k_l = 5.34 \) and \( k_G = 5.72 \left( d/t_w \right)^{1.5} \). Therefore, \( k_l \) is approximately obtained by

\[
k_l = \frac{30.54 \left( d/t_w \right)^{1.5}}{5.34 \left( d/t_w \right)^{-1.5} + 5.72 \left( w/h_w \right)^2}
\]

Considering \( w/h_w \leq 0.2 \) and \( d/t_w \geq 10 \) for actual bridges, it is found that the maximum difference between Eqs. (10) and (11) is 5.1%. Thus, Eq. (11) can be used to calculate the \( k_l \) instead of using Eq. (10).

3. Shear buckling design of trapezoidally corrugated steel webs

The elastic shear buckling strength of corrugated steel webs is controlled by the elastic interactive shear buckling strength. Thus,
the shear buckling parameter of corrugated webs, $\lambda_s$ is defined as

$$\lambda_s = \frac{\tau_y}{\sqrt{\frac{k t_w}{E}}}$$  \hspace{1cm} (12)

where, $\tau_y$ is the shear yielding stress of the webs. Substituting Eq. (9) into Eq. (12), $\lambda_s$ can be expressed as

$$\lambda_s = 1.05 \frac{\tau_y}{k t_w}$$  \hspace{1cm} (13)

Once a determination is made of $\lambda_s$, the shear buckling strength, considering material inelasticity, residual stress, and initial imperfections, can be determined from the buckling curve. In this study, the buckling curve is adopted from the design manual for PC bridges with corrugated steel webs [11]. The buckling curve equations that were used are given by

$$\frac{\tau_y}{\tau_{ys}} = \begin{cases} 1 & \lambda_s < 0.6 \\ 1 - 0.614 (\lambda_s - 0.6) & 0.6 \leq \lambda_s < \sqrt{2} \\ 1/\lambda_s^2 & \lambda_s \geq \sqrt{2}. \end{cases}$$  \hspace{1cm} (14)

Calculation procedure of shear buckling strengths proposed in this study is summarized as follows: Firstly, $\lambda_s$ is calculated from Eq. (13) with Eq. (11). Then, the shear strength of corrugated webs can be determined using Eq. (14) directly. The proposed shear strength of corrugated steel webs is simply calculated because the local and global shear buckling strength is not required and they only need a few material properties ($E$ and $\tau_{ys}$) and geometric properties ($w/t_w$, $d/t_w$, and $h/t_w$).

The shear buckling doesn’t occur when $\lambda_s \leq 0.6$, and the maximum shear capacity can be achieved. Rearranging Eqs. (11) and (13), and $\lambda_s \leq 0.6$, the condition to maximize the shear capacity can be obtained and given by

$$1.10 \left[ \frac{5.34 (d/t_w)^{1.5}}{30.54} \right] \lambda_s \leq 0.36.$$  \hspace{1cm} (15)

Satisfying Eq. (15) will maximize the shear strength of corrugated steel webs without shear buckling. It is recommended that the corrugated steel webs satisfy the Eq. (15) to get maximum shear strength in design.

4. Shear buckling tests of trapezoidally corrugated steel webs

In this section, the experimental study is used to investigate the shear behavior and strength of corrugated steel webs. Firstly, descriptions of profiles and initial imperfections of each test specimen are summarized followed by descriptions of the test setup. The test results are then reported.

4.1. Profiles and initial imperfections of the test specimen

Three different test specimens (MI2–MI4 specimen) were used for this study and the profiles of the specimens are summarized in Table 2. In Table 2, $b_f$ is the width of the flange; $t_f$ is the thickness of the flange; and $L$ is the length of the specimen. All specimens have the same height of $h_w = 2000$ mm, web thickness of $t_w = 4$ mm, flange width of $b_f = 300$ mm, and flange thickness of $t_f = 30$ mm. However, the specimens have different corrugation profiles and a different shear buckling parameter, $\lambda_{sy}$ in the inelastic region, $(0.6 \leq \lambda_s < \sqrt{2})$. From material tests, the yield stress of the corrugated steel web was 296 MPa.

Fig. 4 shows an illustration of the dimensions of a test specimen (MI4 specimen). The load was applied as a one-point loading. In order to prevent bearing failure of the web due to concentrated loads, bearing stiffeners that have a thickness $= 25$ mm were attached at the supports and the loading points. Between the loading stiffeners, a $15$ mm plate was installed to induce the shear deformation of the corrugated steel plate.

The buckling strength is sensitive to the magnitudes of the out-of-plane initial imperfections. Initial imperfections were measured using a laser range finder. The tolerance of the measuring apparatus is ±1 mm. Fig. 5 represents the initial imperfection...
Fig. 5. Initial imperfections of the test specimen (MI2).

Table 3
Minimum, maximum, and average corrugation depth $d$ of test specimen (Unit:mm)

<table>
<thead>
<tr>
<th>Model</th>
<th>$d_{\text{min}}$</th>
<th>$d_{\text{max}}$</th>
<th>$d_{\text{avg}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MI2</td>
<td>68</td>
<td>75</td>
<td>72</td>
</tr>
<tr>
<td>MI3</td>
<td>47</td>
<td>56</td>
<td>51</td>
</tr>
<tr>
<td>MI4</td>
<td>61</td>
<td>71</td>
<td>66</td>
</tr>
</tbody>
</table>

Table 3 shows the minimum, maximum, and average corrugation depth $d$ of the test specimens. The geometric parameters, $d/t_w$ and $w/h_w$, affect the buckling characteristics of corrugated steel webs. In particular, the corrugation depth, $d$ is sensitive to initial imperfections. The maximum, minimum, and average corrugation depths were calculated for all test specimens and are listed in Table 3.

The maximum magnitude of the initial imperfection is 17.9 mm, which was observed close to the right part of the upper flange.

The relationships between shear stress and strain are presented in Fig. 8. The $x$ axis denotes the shear strain, $\gamma$ while the $y$ axis represents the shear stress $\tau$ defined as $\tau = P/(t_w h_w) (L_1/L_2)$ where $P$ is the applied load; and $L_1$ and $L_2$ are shown in Fig. 8. The values of $L_1/L_2$ are 0.534, 0.530, and 0.530 for MI2, MI3, and MI4, respectively. Strain rosettes (M1–M4) were installed at half-height and detail information of the location is shown in Fig. 8. It can be seen that shear strain is uniform through the entire web and that the corrugated steel webs are under pure shear states. The solid lines in Fig. 8 represent the relationship of $\tau = G_{\text{co}} \gamma$, where, $G_{\text{co}}$ is the shear modulus of the corrugated plate. Samanta and Mukhopadhyay [13] proposed that $G_{\text{co}}$ is given by

$$G_{\text{co}} = \frac{(a + b) E}{(a + c) 2 (1 + \nu)} = \eta G$$

(16)
where $\eta$ is the length reduction factor and the value of $\eta$ is less than 1. Therefore, $G_{co}$ is smaller than $G$. It is found that the relationship of $\tau = G_{co}\gamma$ agrees well with the test results.

The variations in shear stress, $\tau$, with vertical displacement, $\delta$, which was measured at the mid-span of the specimen, are shown in Fig. 9. All specimens failed suddenly due to shear buckling, and the shear stresses dramatically decreased. The buckling was then extended to the adjacent fold. This repeated sequence of loading and web buckling resulted in the jagged post-buckling curve, as shown in Fig. 9. However, the post-buckling strength of the corrugated steel webs is negligible since it is smaller than the initial buckling strength. The shear strengths found in Fig. 9 are 109.2, 105.4, and 131.6 MPa for the MI2, MI3, and MI4 specimens, respectively.

Fig. 10 represents the variations in shear stress with out-of-plane displacements of the test specimens. Out-of-plane displacements increased linearly up to critical shear stresses, following which the displacements increased suddenly. These critical values of the shear stresses coincide with the shear buckling stresses obtained by Fig. 9.

5. Verification of proposed design equations and comparison with results obtained by other researchers

In this section, the proposed shear strengths of shear buckling of corrugated steel webs are verified using the test results and published experimental data. Proposed shear strengths are also compared with those of previous researchers.

The experimental results obtained by Yamazaki [9] and Gil et al. [2] are used for comparison. Yamazaki [9] and Gil et al. [2] reported the results of six tests conducted in Japan and nine tests conducted in South Korea, respectively. Both sets of tests were carried out using large-scale test specimens. Many shear buckling tests of corrugated steel webs have been performed in the United States and Europe. However, most of these shear buckling tests were conducted on a relatively small-scale specimen that had a dimension and plate thickness significantly smaller than that which would be used in actual bridge construction. Therefore, in this study, the experimental results that were conducted in the United States and Europe are excluded from comparison.
Fig. 9. Variations in shear stress $\tau$ with vertical displacement $\delta$.

Fig. 10. Variations in shear stress $\tau$ with out-of-plane displacement.

Fig. 11. Comparison of proposed shear strength with those of test results.

Fig. 12. Comparison of shear strength of previous researchers with those of test results.

Fig. 13. Comparison of shear strength of previous researchers with those of test results.

Fig. 14. Design buckling curve (Driver et al., 2006).

- Test Results (This Study)
- Gil et al. (2005)
- Yamazaki (2001)

Driver et al. [10] suggested the shear strengths of corrugated steel webs. These criteria preclude elastic and inelastic global buckling due to a loss of strength associated with the occurrence of global buckling, and to the low degree of post-buckling strength. Shear strengths of corrugated steel webs proposed by Driver et al. [10] are summarized as follows: They proposed the ratio of the web height to the thickness limit in order to achieve the web yielding, based on global buckling equations, as

$$ h_w / t_w \leq 1.91 \varphi \sqrt{ \frac{E}{f_y} \left( \frac{a}{t_w} \right)^{1.5} F(\theta, \alpha) } $$

(17)

where, with the recommended value of 0.9, is introduced to provide a margin of safety on global buckling. Provided that the web slenderness limit of Eq. (17) is satisfied, the buckling strength of the corrugated steel webs based on the 2nd order interactive shear buckling equations that assume $\tau_{cr,G} = \tau_y$, is given by

$$ \frac{\tau_{cr}}{\tau_y} = \begin{cases} 0.707 \sqrt{1/(1+0.15\lambda_L^2)} & 2.586 < \lambda_L \leq 3.233 \\ \sqrt{1/(1+0.014\lambda_L^4)} & \lambda_L > 3.233 \end{cases} $$

(18)

where $\lambda_L$ is the local buckling slenderness in normalized form given by $\lambda_L = (w/t_w) \sqrt{f_y/E}$.

Fig. 12 shows a comparison between the shear strengths proposed by Driver et al. [10] and those of the test results. Because they preclude the global shear buckling, only a few experimental data (they failed due to local buckling) are shown in Fig. 12. In addition, the maximum shear buckling strengths of corrugated steel webs are limited to 70.7% of the shear yielding stress, $\tau_y$ due to the use of the 2nd order interactive equation that assumes $\tau_{cr,G} = \tau_y$. It tends to be too conservative in its estimation of the shear yielding stress and inelastic buckling strength.

According to the design manual for PC bridges with corrugated steel webs [11], shear strengths of the corrugated steel webs can be determined from Eq. (14). However, the large value between $\sqrt{\tau_y / \tau_{cr,G}}$ and $\sqrt{\tau_y / \tau_{cr,G}}$ is used as a shear buckling parameter $\lambda_s$ because interactive shear buckling is not considered. Fig. 13 shows the comparison of shear strengths from the design manual for PC bridges with corrugated steel webs [11] with those of the test results. It can be seen that the design manual for PC bridges with corrugated steel webs [11] overestimates the shear strength for some experimental data.

Compared with the work of previous researchers [10,11], the shear strength proposed in this study gives a better estimation of the shear strength of corrugated steel webs.
6. Conclusions

This study proposes the shear strength and design criteria of trapezoidally corrugated steel webs. A series of experimental studies were also performed in order to verify the proposed equations. Firstly, the global shear buckling equation was rearranged to take the form of the classical plate buckling equation, and the global shear buckling coefficient, \( k_g \), is suggested as a function of \( d/t_w \), as shown in Eq. (7). The elastic interactive shear buckling formula based on the 1st order interactive equation in elastic states is then proposed as a form of the classical plate buckling equation, as described in Eq. (9), where \( k_I \) is the interactive shear buckling coefficient, which is a function of the geometric parameters of the corrugated steel webs, \( d/t_w \) and \( w/h_w \).

For a practical design, the shear buckling parameter, \( \lambda_s \), is suggested. Then, the shear strength of a corrugated steel web can be determined from the buckling curves for a given \( \lambda_s \). The condition to maximize the shear strength capacity is proposed as shown in Eq. (15), and it is recommended that the corrugated steel webs satisfy the Eq. (15) to get the maximum shear strength in a design.

Tests were conducted on 3 test specimens that have a different \( \lambda_s \) in the inelastic region. It was found that all test specimens are under pure shear and there are negligible post-buckling strengths. Proposed shear strength is verified using the test results of this study and those provided by previous researchers. It can be found that the proposed shear strength provides a good estimation of the shear strength of corrugated steel webs.

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