A Simplified Evaluation in Critical Frequency and Wind Speed to Bridge Deck Flutter

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Abstract

This paper deals with the instability problem of flexible bridge decks under an approaching crosswind flow within framework of bimodal flutter involving fundamental vertical and torsional modes of bridge vibration. Start from simplifying coefficients from terms of the cubic and quartic polynomials derived from singularity conditions of an integral wind-structure impedance matrix and by using the quasi-steady approach, the approximated formula for calculating onset flutter of the aeroelastic bridge system are proposed. A good agreement is obtained comparing predictions on the onset flutter by the proposed and available formulas for several study cases from the existing bridges with difference of structural and aerodynamic characteristics of given bridge deck.

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Keywords: Bridges; Eigenvalue; Flutter; Flutter derivatives; Selberg's formula; Simplified formulations.

1 INTRODUCTION

Owing to large flexibility and low structural damping, long span bridges may be susceptible to an approaching crosswind flow. The aeroelastic interaction between wind flow and bridge can be relied on extraneous-flow-induced, flow-instability-induced and movement-induced excitation mechanisms. The latter mechanism is caused by fluctuating wind forces due to movements of the vibrating structural part. Small deviations from the equilibrium position of the structure induce a re-distribution of impacting wind forces, which further increase the initial disturbances. If these self-excited forces lead to a negative damping threshold, the onset of flutter will occur and this directly induces structural failure, as was the case in the collapse of the Tacoma Narrows Bridge in 1940. Among these various aerodynamic

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instabilities, the coupled flutter, originated by the interaction between the wind and the vertical-torsion oscillations, is the most complex. Early pioneering work on bimodal flutter analysis was presented by Bleich (1949), where stiffness-driven flutter was addressed using airfoil aerodynamics theory. While Selberg (1961) and Rocard (1963) proposed simplified empirical formulas using inertial and dynamical properties of the deck section for estimating critical wind speed that is rigorously applied for flat plate sections. However, to apply the Selberg formula the aerodynamic properties of the real deck section can be taken into account only by using empirical corrective factors (Dyrbye and Hansen, 1997). In this framework aim of the paper is to give a contribution to a simpler description of the bridge flutter problem.

In the present paper, a set of simplified formulations, resulting from simplifying coefficients from terms of the cubic and quartic polynomials derived from singularity conditions of an integral wind-structure impedance matrix and by using the quasi-steady approach, the approximated formula for calculating onset flutter of given bridge section is proposed.

2 THEORETICAL BACKGROUND

Considering only the first vertical and torsional modes of the bridge, of which the natural circular frequencies in still are \( \omega_z \) and \( \omega_\theta \). Only the aerodynamic forces on the bridge, which generally dominate the aerodynamic performance, are the self-excited lift and pitching moment acting on the bridge deck section per unit length are given by (Simiu and Scanlan, 1996).

\[
L^{ae} = \frac{1}{2} \rho V^2 B \left[ KH_1^* \frac{\ddot{z}}{V} + KH_2^* \frac{\dot{B}\dot{\theta}}{V} + K^2 H_3^* \theta + K^2 H_4^* \frac{h}{B} \right]
\]

\[
M^{ae} = \frac{1}{2} \rho V^2 B^2 \left[ KA_1^* \frac{\ddot{z}}{V} + KA_2^* \frac{\dot{B}\dot{\theta}}{V} + K^2 A_3^* \theta + K^2 A_4^* \frac{h}{B} \right]
\]

where \( z \) and \( \theta \) denote the vertical and torsional displacements, \( V \) = mean wind speed, \( B \) = bridge deck (Fig. 1), \( \rho \) = air density, \( K = 2k = \omega B/V \) is the reduced frequency, \( H_i^* \), \( A_i^* \) (\( i = 1,4 \)) = flutter derivatives, which are functions of reduced frequency and can be extracted from the wind-tunnel test.

The governing equations of combined bridge system in terms of the generalized modal coordinates \( \eta \) under approaching crosswind flow are expressed as

\[
M \ddot{\eta} + C \dot{\eta} + K \eta = F^{ae}
\]

where \( M, C, K \) = generalized modal mass, damping and stiffness matrices, respectively; \( F^{ae} \) = the generalized self-excited force vectors and over-dot denotes partial differentiation with respect to time.

Taking the Fourier transform on either side of Equation (3) leading to Equation (4) considered as modal equilibrium equation of motion in frequency domain

\[
\left[ -M \omega^2 + i \left( C - C^{ae} \right) \omega + \left( K - K^{ae} \right) \right] X_\eta(\omega) = 0
\]

where \( i^2 = -1 \), matrices of \( C^{ae} \) and \( K^{ae} \) only contained the coefficients that are required for the vertical and torsional motion and they are normalized by \( \rho B^2 \omega^2 \) and \( \rho B^4 \omega \), respectively of which \( \omega \) is the in-wind frequency dependent on mean wind velocity \( (V) \),
\[ C_{\alpha \epsilon}^{\alpha \epsilon} = \frac{\rho B^2 \omega^2}{2} \begin{bmatrix} H_1^* & BH_2^* \\ BA_1^* & B^2 A_2^* \end{bmatrix}, \quad K_{\alpha \epsilon}^{\alpha \epsilon} = \frac{\rho B^2 \omega^2}{2} \begin{bmatrix} H_4^* & BH_5^* \\ BA_4^* & B^2 A_5^* \end{bmatrix} \] (5a,5b)

Any stability limit can be found through setting the determinant of the coefficient matrix for Equation (4) equal to zero, as a result, it leads to the cubic or quartic polynomials for real or imaginary parts with respect to the in-wind frequency ratio \( \Omega \) as follows,

\[ I_1 \Omega^3 + I_2 \Omega^2 + I_3 \Omega + I_4 = 0 \] (6)

\[ R_1 \Omega^4 + R_2 \Omega^3 + R_3 \Omega^2 + 1 = 0 \] (7)

wherein,

\[ I_1 = \gamma^2 \left[ \frac{1}{2} \left( \lambda_z H_1^* + \lambda_\theta A_2^* \right) + \frac{\lambda_z \lambda_\theta}{4} C_1 \right], I_2 = -2 \left[ \xi_z \left( \frac{\lambda_\theta}{2} A_3^* + \gamma \right) + \gamma^2 \xi_\theta \left( \frac{A_z^*}{2} H_4^* + 1 \right) \right] \] (8a,8b)

\[ I_3 = -0.5 \left( \lambda_z \gamma^2 H_1^* + \lambda_\theta A_2^* \right), I_4 = 2 \left( \xi_z \gamma + \xi_\theta \right) \] (8c,8d)

and

\[ R_1 = \gamma^2 \left[ 1 + \frac{1}{2} \left( \lambda_z H_4^* + \lambda_\theta A_3^* \right) + \frac{\lambda_z \lambda_\theta}{4} C_R \right], R_2 = \gamma \left( \xi_z \lambda_z H_1^* + \xi_\theta \lambda_\theta A_2^* \right) \] (9a,9b)

\[ R_3 = -\left[ 1 + \gamma^2 + 4 \xi \xi_\theta \xi_z + \frac{1}{2} \left( \lambda_z \gamma^2 H_4^* + \lambda_\theta A_3^* \right) \right] \] (9c)

**Fig. 1** Sign convention for the displacements and self-excited forces

with \( C_i = H_i^* A_i - H_i^* A_i^* - H_i^* A_i^* + H_i^* A_i^* \), \( C_\theta = A_i^* H_i^* - A_i^* H_i^* + A_i^* H_i^* - A_i^* H_i^* \); \( \gamma = \omega_\theta / \omega_z \) is the structural frequency ratio; \( \lambda_z = \rho B^2 / \overline{m}_z \) and \( \lambda_\theta = \rho B^3 / \overline{m}_\theta \) represent the non-dimensional mass and the polar moment of inertia, respectively; \( \Omega = \omega_c / \omega_\theta \) is the in-wind frequency ratio for bridge sections prone to coupled flutter and is expressed as follows. It can be seen that the solution of these equations
requires searching for the lowest identical roots with respect to $\Omega$, which do not easily allow practical calculation by hand, from both the fourth and third degree polynomial.

3 Approximate ANALYSIS bimodal flutter

3.1 Uncoupled flutter derivatives based formulas

In framework of using low-level damping hypothesis, generally implied in the modeling of self-excited forces, we can neglect the $I_2, I_4$ and $R_2$ terms in the polynomials with respect to the other terms as the good approximation, besides that coefficients of $0.25\gamma^2\beta C_R$ and $0.25\gamma^2\beta C_I$, combined with coupled and uncoupled flutter are also dropped in $R_1$ and $I_1$ terms (Vu et al., 2010a, 2010b). With this context, Equation (6) and Equation (7) become the new polynomials as follow,

\[ 0 = I_1\Omega^3 + I_2\Omega^2 + I_3\Omega + I_4 = I_1\Omega^3 + I_3\Omega \]  
\[ 0 = R_1\Omega^4 + R_2\Omega^3 + R_3\Omega^2 + 1 \approx R_1\Omega^4 + R_3\Omega^2 + 1 \]

whereby,

\[ R_1 = \gamma^2 \left[ 1 + \frac{1}{2} \left( \lambda_z H_4^* + \lambda_y A_3^* \right) + \frac{\lambda_y^2}{4} C_R \right] \approx \gamma^2 \left[ 1 + \frac{1}{2} \left( \lambda_z H_4^* + \lambda_y A_3^* \right) \right] \]  
\[ I_1 = \gamma^2 \left[ \frac{1}{2} \left( \lambda_z H_1^* + \lambda_y A_2^* \right) + \frac{\lambda_z^2}{4} C_I \right] \approx \frac{\gamma^2}{2} \left( \lambda_z H_1^* + \lambda_y A_2^* \right) \]

Therefore, Equation (10) can be rewritten as,

\[ \Omega^2 = -\frac{I_3}{I_1} = \frac{\lambda_z \gamma^2 H_1^* + \lambda_y A_2^*}{\gamma^2 \left( \lambda_z H_1^* + \lambda_y A_2^* \right)} \]

Substituting Equation (14) into Equation (11) while taking into account Equation (9c) and Equation (12), and after rearranging some terms, leads to the simple equation as follows,

\[ \left( 1 - \gamma^2 \right) \lambda_z \lambda_y \left[ \left( H_1^* \lambda_z + A_2^* \lambda_y \right) \left( A_3^* H_4^* - H_4^* A_3^* \right) + 2 H_1^* A_2^* \left( 1 - \gamma^2 \right) \right] + 8 \gamma \zeta \lambda_y \left( A_2^* \lambda_y + H_1^* \lambda_z \gamma^2 \right) \times \left( A_2^* \lambda_y + H_1^* \lambda_z \right) = 0 \]

Equation (14) and Equation (15) consist of uncouple flutter derivative using to estimate the approximated solutions of the critical frequency and reduced wind speed of the combined bridge system. Its pragmatic feature is that it is able to apply well in the case of cross sections prone to whether vertical, torsional flutter or coupled flutter (Vu et al., 2010a, 2010b).
3.2 Further simplified formulations

The in-wind resonance frequency and critical wind speed at an incipient flutter can take the form as

$$\omega_r = \frac{2V_{cr} k}{B}, \quad V_{cr} = \frac{B\omega_r}{2k}$$  \hspace{1cm} (16a,16b)

Substitute equation (16a) into equation (14) leading to

$$\left(\lambda_2 \gamma^2 H_1^* + \lambda_\theta A_2^*\right) = \gamma^2 \left(\lambda_2 H_1^* + \lambda_\theta A_2^*\right) \frac{4V_{cr}^2 k^2}{B^2 \omega_\theta^2}$$  \hspace{1cm} (17)

At this point, substitute equation (17) into equation (15), after rearranging terms, we finally find,

$$V_{cr} = \omega_\theta B \chi \left(1 - \frac{1}{\gamma^2}\right) \frac{2}{\left(\lambda_2 \lambda_\theta\right)^{0.5}}$$  \hspace{1cm} (18)

where,

$$\chi = \sqrt{4k^2 \left(\lambda_2 H_1^* + \lambda_\theta A_2^*\right) \left[\left(\gamma^2 - 1\right) \lambda_2 \lambda_\theta \left(A_3 H_1^* - A_1^* H_4^*\right) - 8\gamma \xi_2 \zeta_\theta \left(H_1^* + \lambda_\theta A_2^*\right)\right]}$$  \hspace{1cm} (19)

At large wind speeds, the aerodynamic loads produced on the section can be approximately represented by the steady flow loads, which do not depend on the reduced frequency. In this regard, the derivatives \(H_1^*, H_4^*\) become proportional to \(1/K\), while the derivatives \(A_4^*\) and \(A_1^*\) become proportional to \(1/K^2\) (Como et al. 2002): \(H_1^* = -h_1/K\); \(H_4^* = -h_4/K\) and \(A_4^* = -a_4/K\); \(A_1^* = a_1/K^2\), wherein \(h_1, h_4, a_2, a_3\): positive constants, will be evaluated by inspection of the diagrams of the aerodynamic functions \(H_1^*, H_4^*, A_2^*\) and \(A_3^*\). Next, the assumption of quasi-steady state was applied for the equation (18).

$$V_r = \lim_{K \to 0} \left[2\pi f_\theta B \theta \left(1 - \frac{1}{\gamma^2}\right) \frac{2}{\lambda_2 \lambda_\theta}\right] = 2\pi f_\theta B \left(1 - \frac{1}{\gamma^2}\right) \frac{2}{a_3 \left(\lambda_2 - \frac{h_1}{a_2} + \lambda_\theta\right)}$$  \hspace{1cm} (20)

It is clear that equation (20) has a structure very similar to the well-known Selberg formula. It also showed that role of the flutter derivatives of \(H_1^*, A_2^*\) and \(A_3^*\) in coupled flutter of the bridge occurring at large reduced wind speed.

Table 1: Geometric and dynamics properties of the different bridges

<table>
<thead>
<tr>
<th>Case</th>
<th>Bridge</th>
<th>B(m)</th>
<th>(f_z)</th>
<th>(\gamma)</th>
<th>(m_z)</th>
<th>(m_\gamma)</th>
<th>(\lambda_2)</th>
<th>(\lambda_\theta)</th>
<th>(h_1/a_2)</th>
<th>(a_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Great Belt</td>
<td>31.0</td>
<td>0.099</td>
<td>2.75</td>
<td>22,700</td>
<td>2,470,000</td>
<td>0.05</td>
<td>0.5</td>
<td>11</td>
<td>0.9</td>
</tr>
<tr>
<td>2</td>
<td>2nd Geo-Germ</td>
<td>16.9</td>
<td>0.185</td>
<td>2.99</td>
<td>11,699</td>
<td>295,250</td>
<td>0.03</td>
<td>0.3</td>
<td>16.9</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>2TF</td>
<td>30.0</td>
<td>0.056</td>
<td>1.72</td>
<td>49,000</td>
<td>7,087,000</td>
<td>0.02</td>
<td>0.1</td>
<td>4.9</td>
<td>0.7</td>
</tr>
</tbody>
</table>
4 Numerical validations

In this section, in order to validate the accuracy and effectiveness of the proposed formula, three case studies are taken into account, two of which that are reported in Table 3 originate from the existing bridges: case study 1 for the 2nd Geo-Germ Bridge (Larsen 2002) and case study 2 for the Great Belt Bridge (Larsen 1993, Nissen 2004). Theirs available experimental wind tunnel test evidences are used as benchmarks. Last slotted girder bridge section of 2TF (Matsumoto et al., 2004) is often used in super long bridges. The geometric and aerodynamic properties of these case studies involved cross sections prone to the coupled-mode flutter listed in Table 1. The results of critical wind speeds calculated by approximated formula are compared with those of CEA, while approximated results calculated from Chen and Kareem (2007), Bartoli and Mannini (2008) formulas are regarded as reference solutions.

Finally, all results of these formulas are gathered in the Table 2. From the results, it can be concluded that proposed formulas that consists of approximate formulations of only there uncouple flutter derivatives show results that are close to those given by the traditional CEA. Furthermore, the flutter velocity obtained by equation (25) to the Great Belt Bridge is \( V_c = 71.81 \text{ m/s} \). This result is only a bit lower than the value of 74.3 m/s, evaluated by wind tunnel test (Larsen 1993, Nissen 2004). Likewise, the value predicted by equation (25) for the flutter velocity in case of 2nd Geo-Germ Bridge is \( V_c = 127.16 \text{ m/s} \) in good agreement with the flutter speed of the bridge that ranges between 130 m/s - 140 m/s, according to wind tunnel tests (Larsen 2002).

Table 2: Results for coupled-mode flutter simulations

<table>
<thead>
<tr>
<th>Analysis methods</th>
<th>Flutter Error</th>
<th>Case Study 1</th>
<th>Case Study 2</th>
<th>Case Study 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present ( V_c )</td>
<td>71.81</td>
<td>127.16</td>
<td>48.15</td>
<td></td>
</tr>
<tr>
<td>( \Delta V_c ) (%)</td>
<td>-2.09</td>
<td>-3.20</td>
<td>1.60</td>
<td></td>
</tr>
<tr>
<td>Selberg, 1963 ( V_c )</td>
<td>73.28</td>
<td>101.91</td>
<td>36.50</td>
<td></td>
</tr>
<tr>
<td>Chen et al., 2007 ( V_c )</td>
<td>75.33</td>
<td>131.23</td>
<td>47.38</td>
<td></td>
</tr>
<tr>
<td>Bartoli et al., 2008 ( V_c )</td>
<td>53.7</td>
<td>131.98</td>
<td>47.06</td>
<td></td>
</tr>
<tr>
<td>CEA ( V_c )</td>
<td>73.31</td>
<td>131.23</td>
<td>47.38</td>
<td></td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

The proposed formula estimates with good approximation flutter velocity of long span bridges in the framework of bimodal flutter with using assumption of quasi-stationary approach. These bridges characterized by structure frequency ratios far from unity and prone to coupled flutter at the large of the reduced wind speed. The flutter speeds calculated by proposed formula for existing bridge such as the Great Belt Bridge, the 2nd Geo-Germ Bridge or bridges with slotted girder sections are agreement with those obtained for these bridges by wind tunnel techniques or by complex eigenvalue analysis.

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