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Lateral-torsional buckling of discretely-braced i-girder

H. E. LEEa*, C. T. NGUYENb†, J.H Moonc and H. S. JOOd

a School of Civil, Environmetal & Architectural Engineering, Korea University (helee@korea.ac.kr)
b School of Civil, Environmetal & Architectural Engineering, Korea University (canhtuan@korea.ac.kr)
c Civil & Architectural Engineering, University of Washington (deadalive@gmail.com)
d School of Civil, Environment & Architectural Engineering, Korea University (obladia@korea.ac.kr)

Abstract

This paper presents an analytical solution for lateral-torsional buckling strength and stiffness requirements of I-girders with discrete torsional bracings subjected to non-uniform bending. The critical moment and torsional stiffness requirement are derived by using an energy method for an arbitrary number of bracing points. An equivalent moment factor is introduced to evaluate buckling moments of fully braced I-girders. From the results, it is found that proposed solutions agree well with results of finite element analyses regardless of the number of bracing points, while results for the equivalent continuous brace stiffness concept are not suitable for multiple discrete torsionally-braced beams.

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Keywords: Lateral-torsional buckling; Torsional bracing; Stiffness requirement; Energy method.

1. INTRODUCTION

Torsional bracing systems are widely used to enhance durability and performance of steel I-girder bridges. A buckling strength of an I-girder can be increased by using an adequate bracing system. Typical torsional bracing types are shown in figure 1. Several studies were conducted on lateral-torsional buckling of an I-girder with torsional bracing system under uniform. Equations were proposed to evaluate a lateral-torsional buckling moment including a contribution of bracing stiffness (Taylor and Ojalvo 1966; Trahair 1993; Yura 2001; Nethercot 1973; Mutton and Trahair 1973; Nguyen et al. 2010). It can be realized that all aforementioned studies are limited to the buckling of beams with torsional bracing under...
uniform bending. Therefore, more intensive studies on I-girders with multiple torsional braces are necessary for bracing design of I-girder bridges.

The objective of this study is to provide an analytical solution for an I-girder with discrete torsional bracing under non-uniform loading conditions. Firstly, an energy method is introduced to derive the buckling moment by using Rayleigh-Ritz method (Chen and Lui 1987; Bazant and Cedolin 1991) for an arbitrary number of bracing points. An elastic buckling moment of an I-girder with discrete torsional braces under non-uniform bending can be obtained by introducing an equivalent moment factor. An equation to evaluate a stiffness requirement is also presented in terms of a buckling moment of a fully braced I-girder corresponding to loading conditions. Results from proposed solutions are then compared with those obtained from previous researchers and those of finite element analyses. From the results, it is found that the proposed solutions are well identical to finite element analyses regardless the number of bracing points, while those from previous studies show limitations on determining the buckling strength as well as the stiffness requirement of I-girders with multiple torsional braces.

2. ELASTIC BUCKLING OF AN I-GIRDER WITH INTERMEDIATE TORSIONAL BRACES UNDER NON-UNIFORM BENDING

2.1. Lateral torsional buckling moment

An I-girder with discrete torsional braces under non-uniform bending is shown in figure 2. The total potential energy of the I-girder including the strain energy of rotational elastic restraints is defined as
\[ \Pi = \int \left[ \frac{1}{2} EI_y (u'')^2 + \frac{1}{2} EI_w (\phi'')^2 + \frac{1}{2} GJ (\phi')^2 - M_x u'' \phi \right] dz + \frac{1}{2} R \sum_{i=1}^{n} \phi_i^2 \]  

where \( E \) is Young’s modulus; \( G \) is the shear modulus of elasticity; \( I_y \) is the 2nd moment of inertia about the \( y \) axis; \( I_w \) is the warping constant; \( J \) is the pure torsional constant; and \( \phi_i \) is the twisting angle of the cross section at bracing points; \( n \) is the number of torsional braces. Bending moment \( M_x \) is described as \( M_x = M_o \zeta(z) \) where \( M_o \) is a maximum moment and \( \zeta(z) \) is proposed as an admissible function. Applying the Rayleigh-Ritz method, the first variation of the total potential energy with respect to \( u_k \) and \( \phi_k \) must vanish so that the stationary conditions are given as \( \partial\Pi / \partial u_k = 0 \) and \( \partial\Pi / \partial \phi_k = 0 \). The critical moments can be obtained from nontrivial solutions corresponding to loading conditions and are expressed as

\[ M_{cr,m} = \sqrt{C_{b,m} \left( \frac{\pi}{L / m} \right)^2 EI_y GJ \left( 1 + \left( \frac{\pi}{L / m} \right)^2 EI_w / GJ \right) + (n+1) \frac{C_{b,(m+1)} R EI_y}{L}} \]  

(a)

for \( m = 1, 2, \ldots, n-1 \) when \( R \leq \Sigma R_m \);

\[ M_{cr,n} = C_{b,n} \frac{\pi}{L} EI_y GJ \left( \alpha_n + \beta_n W^2 - 2 \gamma_n \sqrt{1 + 2 \alpha_n W^2} \right)^2 + \frac{1}{C_{b,n}^2} \left( \frac{LR}{2\pi^2 GJ} \right)^2 + \frac{C_{b,(n+1)} \gamma_n LR}{\pi^2 GJ} \]  

(b)

when \( \Sigma R_m < R \leq R_T \);

\[ M_{cr,(n+1)} = C_{b,(n+1)} \frac{\pi}{L / (n+1)} EI_y GJ \left( 1 + \left( \frac{\pi}{L / (n+1)} \right)^2 EI_w / GJ \right) \]  

(c)

when \( R > R_T \)

where \( W \) is a torsional slenderness which is defined as \( W = (\pi / L)\sqrt{EI_w / GJ} \); \( \alpha_n \), \( \beta_n \), \( \gamma_n \) are functions of the number of bracing points \( n \) and are given as \( \alpha_n = (n+1)^2 + 1 \); \( \beta_n = (n+1)^4 + 6(n+1)^2 + 1 \); \( \gamma_n = n+1 \); \( C_b \) is an equivalent moment factor corresponding to loading conditions and is defined depending on buckling configurations.

2.2. Equivalent moment factor

From the analytical results a modified equivalent moment factor is proposed for a fully braced I-girder under non-uniform bending, and a buckling moment can be calculated as

\[ M_{cr,k} = C_{b,k} M_{ocr,k} \]  

(3)

where \( M_{cr,k} \) is a buckling moment of a beam under non-uniform bending while \( M_{ocr,k} \) is a corresponding buckling moment of a beam under pure bending; and \( C_{b,k} \) is an equivalent moment factor and can be approximated as
\[ C_{b,k} = C_b \left(1 + \frac{\lambda}{k^2}\right) \]  

(4)

where \( C_b \) is a conventional equivalent factor for unbraced beam, \( C_b = 1.35 \) for concentrated load and \( C_b = 1.13 \) for uniform load (Chen and Lui 1987); \( C_{b,k} = C_b \) for an unbraced beam \((k = 1)\); \( \lambda \) is a loading factor defined as \( \lambda = 1.0 \) for concentrated load and \( \lambda = 0.5 \) for a uniform load.

2.3. Elastic torsional stiffness requirement

The bracing stiffness \( \Sigma R_m \) is then obtained summing each incremental stiffness and are given as

\[ \Sigma R_m = \frac{1}{C_{b,n}} \left( \frac{M_{cr,n}^2 - M_{cr}^2}{(n+1)EI_y} \right) L . \]  

(5)

For an I-girder subjected to non-uniform bending, the total stiffness requirement can be similarly evaluated as

\[ R_t = \frac{M_{cr,(n+1)}^2 L_b}{C_{b,(n+1)} EI_y K_w} . \]  

(6)

where \( M_{cr,(n+1)} \) is a buckling moment of a fully braced beam, \( M_{cr,(n+1)} \) is calculated by using equation (2c); \( C_{b,(n+1)} \) is an equivalent moment factor for a fully braced buckling moment, \( C_{b,(n+1)} \) is calculated from equation (4) with \( k = n + 1 \); \( L_b \) is an unbraced length; \( K_w \) is a dimensionless factor which is a function of the number of bracing points \( n \) and the torsional slenderness ratio \( W \) (Nguyen et al. 2010).

3. VERIFICATION, COMPARATIVE STUDY, AND DESIGN RECOMMENDATION

3.1. FEM model description

In this study, finite element analyses are conducted to evaluate the buckling moment by using a FE program BASP (Choo 1987). Several I-girder models are selected based on web and flange proportions to prevent local buckling according to the AISC specification (AISC 2001). Analyzed beams are fully stiffened by transverse stiffeners attached at bracing points with full height and width to avoid cross-section distortions. The thickness of stiffeners is 20mm, and the unbraced lengths of the girders vary from 3350mm to 5350mm. Detailed profiles of analysis models are given in Table 1.
Table 1 Profiles of analysis models

<table>
<thead>
<tr>
<th>Model</th>
<th>IG1</th>
<th>IG2</th>
<th>IG3</th>
<th>IG4</th>
<th>IG5</th>
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<th>IG7</th>
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<tr>
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<td>4650</td>
<td>4850</td>
<td>5000</td>
<td>5150</td>
<td>5350</td>
</tr>
</tbody>
</table>

3.2. Results obtained by previous researchers

The buckling moment of an I-girder with discrete torsional bracings can be obtained basing on the continuous bracing concept (Yura 2001) and given as

$$M_{cr} = \sqrt{\frac{C^2_hM^2_{ocr} + C^2_{b,(n+1)}}{C_T R}} \bar{EI_y}$$  \hspace{1cm} (7)

where $C_T$ is a top-flange modification factor, $C_T = 1.2$ for top-flange loading and $C_T = 1.0$ for centroidal loading; $\bar{R}$ is an equivalent continuous bracing stiffness defined as $\bar{R} = nR / L$.

An elastic stiffness requirement of discrete torsional bracings can be derived by substituting $R_T^* = R / L$ into equation (7). The required stiffness $R_T^*$ is then expressed as

$$R_T^* = \frac{C_T}{C^2_{b,(n+1)}} \left( \frac{M^2_{cr,(n+1)} - C^2_hM^2_{ocr}}{nEI_y} \right)$$  \hspace{1cm} (8)

where $M_{cr,(n+1)}$ is a buckling moment of a beam under non-uniform bending when full bracing is provided. Yura conducted finite element analyses on I-girders with multiple bracings subjected to a concentrated load at midspan and suggested $C_{b,(n+1)} = 1.75$ for a midspan torsional brace $(n = 1)$ and $C_{b,(n+1)} = 1.30$ for three torsional braces $(n = 3)$ (Yura 2001).

3.3. Verification and comparison

Figure 3 show comparisons of buckling moments of fully braced I-girders subjected to concentrated load at midspan. It can be seen that results from this study show good agreement with those provided by previous study (Yura 2001). Buckling moments obtained from all proposed solutions are well matched with those from finite element analyses. Proposed solution in this study can provide a significantly good estimation of buckling moments of fully braced beams regardless the number of bracing points.
Figure 3: Buckling moment of fully braced beams with a midspan torsional brace.

Figure 4 shows variations of buckling moments with increasing torsional stiffness. It is found that results from this study are identical to those from finite element analyses while equation (7) shows overestimations when the torsional stiffness begins to reach the limiting value. These results, therefore, reveal that the equivalent continuous torsional bracing concept is not suitable for a discrete bracing system.

Results of stiffness requirement for beams with multiple torsional brace are given in figure 5. For beams under a concentrated load, the stiffness requirements obtained from equation (8) are almost 12.5% underestimated. Again, equation (6) can be used to calculate the stiffness requirement for beam subjected to a concentrated load and a uniform load. The results from this study are in good agreement with those from finite element analyses.

It is found that the equivalent continuous torsional bracing concept cannot provide sufficient required stiffness. Equation (8) significantly underestimates the stiffness requirements with 60% and 70% comparing to those from finite element analyses for beams subjected to a uniform load. The results prove
that the proposed solution in this study can be applied with a sufficient accuracy to evaluate the stiffness requirement for all cases of loading conditions and an arbitrary number of bracing points along the span.

4. CONCLUSIONS

This paper presents an analytical solution for the LTB strength and stiffness requirement of I-girders with discrete torsional bracings. Firstly, critical moments are derived based on the energy method as shown in equation (8). The required stiffness, which changes the buckling configuration and the total stiffness requirement are then proposed as shown in equations (13) and (14), respectively. The proposed solutions are applicable for beams subjected to a concentrated load and a uniform load with an arbitrary number of bracing points \( n \).

The proposed solutions are compared with the results of the finite element analyses and those obtained by previous researchers. From the comparison results, the proposed equations are successfully verified and it is found that the equivalent continuous brace
stiffness concept is not suitable for calculating the total stiffness requirement $R_T$. In particular, the equivalent continuous brace stiffness concept significantly underestimated the total torsional stiffness requirement for larger numbers of bracing points. Also, the critical moment obtained from the equivalent continuous brace stiffness concept is not well matched with the results of the finite element analyses when $n$ is small, while the solution proposed in this study agrees well with the results of the finite element analyses, regardless of the number of bracing points. The effects of loading conditions are investigated. An equivalent moment factor is suggested to calculate the buckling moment of a fully braced I-girder as given in equation (4) based on the results of numerical investigation.

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