Simulation of Accordion Effect of I-girder with Corrugated Steel Webs

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**Abstract.** The axial stiffness of the corrugated steel plate is negligible by nature of unique geometric characteristics of the plate called as the accordion effect. This unique effect results in high efficiency on post-tensioning. Thus, corrugated steel plate is very suitable for a web of PC-box girder. Recent researches show that strain and stress in sub-panel of corrugated steel webs, that are induced by local bending of the sub-panel, exist even if the axial stiffness is negligible. These strain and stress in sub-panel are important since it might cause fatigue failure of the structure under repeated loading. This study presents the analytical simulation of the accordion effect of the I-girder with corrugated steel webs under pure axial load or bending including the effect of local bending of sub-panel of the corrugated steel webs. Theoretical study and numerical results were combined to develop the simplified equation to evaluate the accordion effect including the effect of local bending of the sub-panel.

**Introduction**

Corrugated steel plates are composed of a series of plane and inclined sub-panels as shown in Fig. 1.

![Corrugated steel plate and schematic view of the accordion effect.](image)

The primary characteristic of the corrugated steel plates is negligible axial stiffness, which is called as the accordion effect [1]. Fig. 1 shows schematic view of the accordion effect of the corrugated steel plate when axial force $N$ is applied. This shape change results in negligible axial strain and stress in the corrugated steel plate because most of the axial deformation comes from this shape change. However, recent researches [2] show that strain and stress due to local bending in sub-panel of corrugated steel webs exist. This strain and stress induced by local bending in sub-panel are important because these strain and stress could cause fatigue problems at interface between steel flange and webs (This interface is usually welded in practice) when the I-girder with corrugated steel webs is subjected to repeated bending.

This study presents an analytical simulation of the accordion effect of I-girder with the corrugated steel webs including the effect of local bending of the sub-panel. The theoretical model of the accordion effect of the corrugated steel web subjected to pure axial load proposed by previous researcher was extended to the case of bending. A series of the parametric study was then conducted to evaluate the theoretical model for accordion effect. From the results, it was found that theoretical model is not appropriate at the interface region between steel flange and webs and modified theoretical model for accordion effect was proposed based on the results of parametric study.
Theoretical Model

Fig. 2 shows the assumed deformed shape of the corrugated steel webs subjected to tension where it is assumed that corrugation angle is not changed so that \( \theta_1 = \beta - \theta_2 \) is satisfied as shown in Fig. 2.

\[
\theta_1 = \cos(\theta_2 - \beta) = \cos(\theta_2)\cos(\beta) + \sin(\theta_2)\sin(\beta).
\]

In Fig. 2, \( M_w \) is local bending moment in panel, and \( \varepsilon_t \) is the strain of the extreme fiber of the webs due to local bending of the panel, respectively. Mori et al. [2] theoretically derived \( \varepsilon_t \) and \( f_t \) as

\[
\varepsilon_t = -\varepsilon_w \frac{a + b}{3a + c} \frac{3t_w}{d}, \text{ and } f_t = \frac{E_s}{1 - \nu^2} \varepsilon_t. \tag{1}
\]

In Eq. 1, \( t_w \) is the thickness of panel, \( f_t \) is the stress of the extreme fiber of the webs due to local bending of the panel, and \( \varepsilon_w \) is normal strain of the corrugated steel web, where \( \varepsilon_w \) is defined as \( \Delta L/L \) (See Fig. 2). From the compatibility condition, \( \varepsilon_w \) is the same as the strain of the flange \( \varepsilon_f \). For pure axial load case, \( \varepsilon_f \) is obtained as \( N/E_s A_f \) where \( N \) is the applied axial load and \( E_s A_f \) is axial stiffness of the flange. Similarly, for bending, \( \varepsilon_f \) can be calculated as \( -yM/E_s I_f \) where \( M \) is the applied moment to the girder, \( y \) is the distance from the neutral axis of the girder to point of interest, and \( E_s I_f \) is the bending stiffness of the flange. Thus, for combined axial-bending loading, strain and stress of the extreme fiber of the webs due to local bending of the panel are obtained as

\[
\varepsilon_t = \left( \frac{yM}{E_s I_f} - \frac{N}{E_s A_f} \right) \frac{a + b}{3a + c} \frac{3t_w}{d}, \text{ and } f_t = \frac{E_s}{1 - \nu^2} \left( \frac{yM}{E_s I_f} - \frac{N}{E_s A_f} \right) \frac{a + b}{3a + c} \frac{3t_w}{d}. \tag{2}
\]

Numerical Simulation & Modified Theoretical Model

Numerical simulation of accordion effect. Numerical simulation was conducted by using general purpose structural analysis program ABAQUS [3] to evaluate the theoretical model.

Fig. 3. Comparison results (Pure axial load case): (a) Out \((y_1 = +t_w/2)\); (b) In \((y_1 = -t_w/2)\).

Fig. 3 shows the comparison results of theoretical stresses with analysis results for I-girder with corrugated steel webs under pure axial load where \( y_1 \) is \(+t_w/2\) and \(-t_w/2\) for out and inside webs (Refer Fig. 2). In Fig. 3, \( y \) axis denotes the normalized height of the girder \( y \). From the Fig.3, it can
be seen that stresses in corrugated steel webs have almost same values with opposite sign, which means that these stresses are induced by local bending of the webs and axial stress in the web is almost zero. On the other hand, stresses in lower and upper flange (where \( \gamma \) is -0.5 and 0.5, respectively), stresses are almost same with same direction. This implies these stresses are come from the axial deformation and local bending of the flange is negligible. Also, it was found that the theoretical model compares analysis results well except the region of interface between the webs and flange. This region was defined as ‘Transition zone’ and the rest part except the transition zone of the corrugated steel webs is defined as ‘Central zone’ in this study.

![Fig. 4. Axial and normal stresses (Pure axial load case):](image)
(a) axial stresses in middle surface; (b) Normal stresses in outer surface.

![Fig. 5. Comparison results (Pure bending case):](image)
(a) Out \( (y_t = +t_w/2) \), (b) In \( (y_t = -t_w/2) \).

![Fig. 6. Axial and normal stresses (Pure bending case):](image)
(a) Axial stresses in middle surface; (b) Normal stresses in outer surface.

Stresses in middle surface, which represent the axial stress, and normal stress induced by local bending were separated and plotted as shown in Fig. 4. It can be seen that axial stresses in middle surface of the corrugated steel webs is almost zero. However, axial stresses in middle surface in corrugated steel webs dramatically increased in transition zone as shown in Fig. 4(a), and the sign of the bending normal stress in outer surface is changed as shown in Fig. 4(b). This is because that corrugated steel webs in transition zone undergo axial strain with opposite bending. This opposite bending is also confirmed by deformed shape of the I-girder with corrugated steel webs shown in result of numerical simulation. Taken as whole, following conclusions are made: (1) axial force is...
almost resisted by lower and upper flange, and axial stresses in central zone of the corrugated steel webs are negligible. (2) axial stresses are considerably increased in transition zone of the corrugated steel webs, and the direction of the local bending in this region is opposite to that of central zone. Fig. 5 shows stresses in longitudinal direction of the girder subjected pure bending. Similar with pure axial load case, stresses in out and inside of the corrugated steel web have almost same magnitude with opposite direction. Thus, it can be expected that axial stresses in corrugated steel webs in negligible. The theoretical model described in previous section agrees well with analysis results except transition zone as shown in Figs. 5(a) and (b). This discrepancy is caused by corrugated steel webs in transition zone undergo significant axial stain and opposite bending to central zone. The axial and normal stresses in the girder were plotted and the results are shown Fig 6. Fig. 6(a) and (b) show axial strain in middle surface and bending normal stress in outer surface, respectively. Axial strain in corrugated steel webs is negligibly small. Thus, it can be known that bending moment is almost resisted by axial force in lower and upper flange. Normal stresses induced by local bending of the corrugated steel web is linearly increased as shown in Fig. 6(b) and it is expected since \( f_t \) is a linear function distance from the neutral axis of the girder to point of interest \( y \) as described in Eq. (2). Equation (2) is not appropriate in transition zone as shown in Fig. 6(b) and it is needed to be studied in depth to fully understand the accordion effect of the I-girder with corrugated steel webs.

**Modified theoretical model for accordion effect.** As discussed in previous section, theoretical model for accordion effect of the I-girder with corrugated steel webs is not appropriate in transition zone. This is mainly contributed by neglecting the behavior of interface between the flange and webs during the derivation of Eq. (2). The behavior of the transition zone was studied in depth and modified theoretical model for accordion effect of the I-girder with corrugated steel webs was developed herein though a series of parametric study. Firstly, the maximum bending normal stress in transition zone \( f_{tr} \) was evaluated for various parameters and it was found that \( f_{tr} \) is significantly affected by \( t_w/a \). From the results of parametric study, the equation to estimate \( f_{tr} \) is obtained by

\[
\begin{align*}
    f_{tra} &= 0.2 \left( \frac{t_w}{a} \right)^{-0.6} \frac{E_s}{1-\nu^2} \left( \frac{N}{E_f A_f} \right) \frac{a + b \ 3t_w}{3a + c \ d} \quad \text{(a)} \\
    f_{trb} &= -0.12 \left( \frac{t_w}{a} \right)^{-0.7} \frac{E_s}{1-\nu^2} \left( \frac{h_w M}{2E_f I_f} \right) \frac{a + b \ 3t_w}{3a + c \ d} \quad \text{(b)} \\
    f_{tr} &= f_{tra} + f_{trb} \quad \text{(c)}.
\end{align*}
\]

where \( f_{tra} \) and \( f_{trb} \) are the maximum bending normal stress in transition zone in the case of pure axial and bending case, respectively. To obtain the length of transition zone, the point to start to increase the axial strain significantly in the corrugated steel webs is defined as \( \gamma_a \) and \( \gamma_b \) for pure axial and bending case, respectively. where \( \gamma_a \) and \( \gamma_b \) are normalized height by total height of the girder and \( \gamma_a \) and \( \gamma_b \) are equal to -0.5, 0, and 0.5 for lower flange, center of the girder, and upper flange, respectively. From the results of parametric study, it was found that \( \gamma_a \) and \( \gamma_b \) is affected by \( h_w/a \) and \( \gamma_a \) and \( \gamma_b \) are increased with increasing \( h_w/a \), which means transition zone can be reduced by increasing \( h_w/a \). On average, \( \gamma_a \) and \( \gamma_b \) were 0.4 and 0.35, respectively.

Once \( \gamma_a, \gamma_b, \) and \( f_{tr} \) are known, \( f_t \) in Eq. (2) can be modified by assuming linear variation of the stress, and modified equations can be expressed as

\[
\begin{align*}
    f_t &= \frac{E_s}{1-\nu^2} \left( \frac{y M}{E_f I_f} - \frac{N}{E_f A_f} \right) \frac{a + b \ 3t_w}{3a + c \ d} \quad \text{when } 0 \leq |\gamma| \leq 0.35 \quad \text{(a)} \\
    f_t &= f_r + \left( f_{tr} - f_t \right) |\gamma|/0.35 / 0.15 \quad \text{when } 0.35 \leq |\gamma| \leq 0.4 \quad \text{(b)} \\
    f_t &= f_r + \left( f_{tr} - f_t \right) |\gamma|/0.35 / 0.15 + f_t \left| |\gamma| - 0.4 \right|/0.1 \quad \text{when } 0.4 \leq |\gamma| \quad \text{(c)}.
\end{align*}
\]
where $f_r$ is the stress obtained from Eq. (4a) assuming $\gamma=0.35$ and $f_f$ is the stress at the flange and defined as $N/A_f M_y/I_f$ where $y$ is equal to $h_w/2$ and $-h_w/2$ for upper and lower flange.

![Axial stress in middle surface](image1.png) ![Normal stress in outer surface](image2.png)

Fig. 7. Comparison of Eq. (4) with analysis (Pure bending case): (a) axial stresses in middle surface; (b) Normal stresses in outer surface.

Fig. 7 shows the comparison of stress computed by using Eq. (4) with that from numerical simulation and it can be seen the modified equation provide accurate estimation of the stress in transition zone as well as central zone.

Conclusions

In this study, an analytical simulation of the accordion effect of I-girder with the corrugated steel webs including the effect of local bending of the sub-panel was studied. The theoretical was evaluated by comparing with numerical simulation. From the results, it was found that (1) axial force is almost resisted by lower and upper flange, and axial stresses in central zone of the corrugated steel webs are negligible. (2) axial stresses are considerably increased in transition zone of the corrugated steel webs, and the direction of the local bending in this region is opposite to that of central zone. (3) Thus, theoretical model is not appropriate at transition zone. From the parametric study, modified theoretical model for accordion effect of the I-girder with corrugated steel webs are proposed as Eq. (4).

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