Moment gradient correction factor and inelastic flexural–torsional buckling of I-girder with corrugated steel webs

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A B S T R A C T
Corrugated steel plates have several advantages such as high resistance for shear without stiffeners, minimization of welding process, and high fatigue resistance. To take advantage of these benefits, several researchers have attempted to use corrugated steel plate as a web for I-girders. The flexural–torsional buckling is the major design aspect of such I-girders. However, the flexural–torsional buckling of the I-girder with corrugated steel webs still needs to be investigated especially for a real loading condition such as non-uniform bending. This paper investigated the flexural–torsional buckling strength of an I-girder with corrugated steel webs under linear moment gradient by using finite element analysis. From the results, it was found that the buckling behavior of the I-girder with corrugated steel webs differed depending on the number of periods of the corrugation. Also, a simple equation for the moment gradient correction factor for the I-girder with corrugated steel webs was suggested. The inelastic flexural–torsional buckling strength of the I-girder with corrugated steel webs was then discussed based on current design equations for ordinary I-girders and inelastic finite element analysis considering initial imperfection and residual stresses.

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1. Introduction

The trapezoidally corrugated plates are composed of a series of plane and inclined sub-panels. Corrugated steel plates have been used as building and bridge components due to several advantages such as high shear resistance and out-of-plane stiffness. Research on the corrugated plates was initiated by Easley and McFarland [1]. Since then, numerous theoretical and experimental researches on the characteristics and strength of corrugated steel plates have been studied. The primary characteristic of the corrugated steel plates is negligible bending capacity, which is called the accordion effect. Thus, corrugated steel webs carry only shear forces and the flanges carry the bending moment so that the efficiency of pre-stressing is enhanced. To take advantage of this characteristic, pre-stressed concrete box girder bridges with a corrugated steel web have been constructed in France and Japan [2].

Recently, several researchers [3–6] have attempted to use corrugated plates as the web of I-girders, as shown in Fig. 1. This can overcome the disadvantages of conventional stiffened flat webs such as web instability due to bending stress and also provide high fatigue resistance by minimization of the welding process. Most of the previous studies focused on the shear buckling of corrugated steel webs [1,2,7–9] because the corrugated steel webs fail due to shear buckling or yielding, and three different shear buckling modes (local, global, and interactive) are possible depending on the geometric characteristics of corrugated steel webs. However, for I-girders with corrugated steel webs, the bending is the major design aspect, and it is crucial to thoroughly understand the flexural–torsional buckling of the I-girder with corrugated steel webs.


Despite these many studies, the flexural–torsional buckling of the I-girder with corrugated steel webs still needs to be investigated especially for a real loading condition such as non-uniform bending. In this paper, the focus was on the flexural–torsional buckling of the I-girder with corrugated steel webs under a linear...
moment gradient. First, background theory on the flexural-torsional buckling of the I-girder with corrugated steel webs under uniform bending was introduced. The elastic flexural-torsional buckling of the I-girder with corrugated steel webs under linear moment gradient was then investigated by using finite element analysis. A simple equation for the moment gradient correction factor was then proposed. Finally, the inelastic flexural-torsional buckling strength of the I-girder with corrugated steel webs under linear moment gradient was investigated based on current design equations for ordinary I-girder and inelastic finite element analysis considering initial imperfection and residual stresses.

2. Background theory

2.1. Flexural-torsional buckling of I-girder with corrugated steel webs under uniform bending and section properties

Fig. 2 shows the profiles of the I-girder with corrugated steel webs used in this study. The x, y, and z axes shown in Fig. 2(a) represent the global coordinates of the I-girder. Fig. 2(b) shows a cross-section of the I-girder with corrugated steel webs, where \( h_w \) is the height of the web, \( b_f \) is the width of the upper and lower flanges, \( t_f \) is the thickness of the flanges, \( o \) is the center of the upper and lower flanges, and \( d \) is the depth of corrugation which varies along the longitudinal direction. The one period of corrugation \((n=1)\) is shown in Fig. 2(c) where \( a \) is the length of flat panel, \( b \) is the projection length of the inclined panel, \( c \) is the length of inclined panel, \( \theta \) is the corrugation angle, \( t_w \) is the thickness of the corrugated steel webs, and \( d_{\text{max}} \) is the maximum depth of corrugation.

Moon et al. [6] proposed the flexural-torsional buckling strength of I-girder with corrugated steel webs under uniform bending based on the buckling equation for the I-girder with flat webs. The elastic flexural-torsional buckling strength of the I-girder with corrugated steel webs under uniform bending \( M_{\text{ocr}} \) can be expressed as

\[
M_{\text{ocr}} = \frac{\pi}{l} \sqrt{\frac{E I_{y,co} G_{co}}{J_{co}}} \sqrt{1 + W^2}, \quad W = \frac{\pi}{l} \sqrt{\frac{E C w_{co}}{G_{co} J_{co}}}
\]  

(1)

where \( l \) is the length of the I-girder, \( W \) is the torsional slenderness which represents the effect of warping torsional stiffness, \( E \) is the Young’s modulus, \( I_{y,co} \) is the second moment of inertia about the weak axis which is given by \( I_{y,co} = \frac{tfb_f}{3} \) assuming no contribution of the web to the flexural behavior due to accordion effect [10], and \( G_{co} \) is the shear modulus of the corrugated plate and is defined as [14]

\[
G_{co} = \frac{a + b}{a + c} G = \eta G
\]  

(2)

In Eq. (2), \( \eta \) is the ratio of the projection length to the actual length of corrugation, and \( G \) is the shear modulus of the flat plate. Generally, \( \eta \) is varied from 0.8 to 0.9 for practical corrugation profiles. The pure torsional constant of the I-girder with corrugated steel webs \( J_{co} \) in Eq. (1) is the same as that of the I-girder with flat webs and can be expressed as [6,11]

\[
J_{co} = \frac{1}{3} (2bft_f^2 + hw t_w^2)
\]  

(3)
The warping constant of the I-girder with corrugated steel webs \(C_{w,co}\) considerably differs to that of the I-girder with flat webs. \(C_{w,co}\) increases with increasing corrugation depth \(d\) [6,11,13]. Moon et al. [6] theoretically derived \(C_{w,co}\) and proposed \(C_{w,co}\) is given by

\[
C_{w,co} = \frac{1}{3} \sum c_{m0} \left(W_n^2 + W_{ni}^2 + W_{n0}^2\right) \frac{t_i t_j}{t_m}
\]

in which,

\[
W_n = \frac{2b_j^2 h_{lw} t_f + b_j^2 h_{lw} t_w}{8b_j t_f + 4h_{lw} t_w}
\]

\[
W_{n2} = \frac{2b_j^2 h_{lw} t_f + b_j^2 h_{lw} t_w}{8b_j t_f + 4h_{lw} t_w} - \left(\frac{b_j}{4} - \frac{t_{avg}}{2}\right)h_{lw},
\]

\[
W_{n3} = \frac{2b_j^2 h_{lw} t_f + b_j^2 h_{lw} t_w}{8b_j t_f + 4h_{lw} t_w} - \frac{b_j}{4}h_{lw},
\]

\[
W_{n4} = \frac{2b_j^2 h_{lw} t_f + b_j^2 h_{lw} t_w}{8b_j t_f + 4h_{lw} t_w} - \frac{1}{2}b_j h_{lw},
\]

\[
W_{n5} = W_{n4}. \quad W_{n6} = W_{n1},
\]

\[
d_{avg} = \frac{(2a + b)}{2(a + b)}d_{max}.
\]

In Eq. (4), \(W_n\) is the normalized unit warping at point \(i\) of any element \((i-j)\), \(t_j\) is length of plate element, \(t_i\) is thickness of plate element, and \(d_{avg}\) is average corrugation depth. The intersection point of the plate element of the I-girder with corrugated steel webs is shown in Fig. 2(b). Thus, \(C_{w,co}\) can be calculated as follows: (a) calculation of \(d_{avg}\) using Eq. (5); (b) evaluation of \(W_{n}\) using Eq. (5) with \(d_{avg}\) obtained in step (a); and (c) determination of \(C_{w,co}\) using Eq. (4) with \(W_{n}\) obtained in step (b).

### 2.2. Flange transverse bending of I-girder with corrugated steel webs

Fig. 3(a) and (b) show the shear flow and shear force, respectively, acting on the cross-section of the I-girder with corrugated steel webs under bending action [6]. The unbalanced shear force on the flange \(V_f\) is generated because the summation of shear flow acting on the flange is not equal to zero, as shown in Fig. 3(a) and (b). \(V_f\) is equal to zero when the corrugation depth is equal to zero. Thus, unbalanced shear force on the flange \(V_f\) and shear force on the web produce twist moment about the center of the upper and lower flanges \(a\). Finally, the I-girder with corrugated steel webs twists out-of-plane simultaneously as it deflects in-plane when the girder is subjected to non-uniform bending moment [3,4,6].

Fig. 4(a) shows the flange transverse bending mechanism. The constant shear force is generated when the beam is subjected to two different end moments, \(M_1\) and \(M_2\) as shown in Fig. 4(a). The generated shear force produces the flange transverse bending moment \(M_{bf}\) due to the eccentricity of the webs [3,4]. Flanges are deformed in the out-of-plane direction with a single curvature when the I-girder with corrugated steel webs has an even number of half corrugations \((n=1, 2, \ldots)\), while double curvature bending occurs when the girder has an odd number of half corrugations \((n=1.5, 2.5, 3.5, \ldots)\) as shown in Fig. 4(a).

Fig. 4(b) shows the example of the deformed shape of the I-girder with corrugated steel webs under unequal end moment where the length of flat panel \(a\) is 300 mm, the length of inclined panel \(c\) is 300 mm, the corrugation angle \(\theta\) is 36.8°, the height of the web \(h_w\) is 1500 mm, the thickness of the corrugated steel webs \(t_w\) is 15 mm, the width of the upper and lower flanges \(b_f\) is 500 mm, and the thickness of the flanges \(t_f\) is 50 mm. It can be seen that the flange deformed with single curvature when the periods of corrugation \(n\) is equal to 10, while, for \(n=10.5\), the flange deformed with double curvature so that the out-of-plane deformation at the center of the I-girder is equal to zero. Because of these different behaviors depending on numbers of periods of corrugation, the effects of the out-of-plane deformation of the flanges, which occurs prior to the buckling, on the flexural-torsional buckling behavior should be considered.

### 3. Elastic flexural–torsional buckling behavior and moment gradient correction factor for I-girder with corrugated steel webs

#### 3.1. Description of finite element models

In this study, the non-linear finite element analyses were performed by using the structural analysis program ABAQUS [15] to investigate the flexural–torsional buckling of the I-girder with corrugated steel webs under linear moment gradient. The 4-node shell element with reduced integration element (54R) was used to model the I-girder. Table 1 shows the profiles of analysis models. The dimensions of analyzed models were based on the profiles of existing bridges with corrugated steel webs. Two different corrugation profiles were used in this study. Corrugation profile 1 (C.P.1) and 2 (C.P.2) have the same dimensions, except the projection length of the inclined panel \(b\); C.P.1 and 2 therefore have a different corrugation angle \(\theta\). In Table 1, \(l_n\) is the length of one period of corrugation. Thus, the total length of the I-girder \(l\) can be calculated by multiplying the number of periods of the corrugation \(n(=l_n \times n)\). For each corrugation profile listed in Table 1, \(n\) is set as 10 and 10.5 to study the effect of the number of corrugation periods on the flexural–torsional buckling behavior. Furthermore, nine different ratios of the end moment \(M_3/M_2\) (\(-1, -0.75, -0.5, 0.5, 0.75, 1\)) were used to investigate the effect of linear moment gradient.

To prevent the local buckling of the flanges prior to the flexural–torsional buckling, the thickness of the flange was selected to satisfy the Class 1 section in Eurocode 3 [16] and compact section in AISC [17]. The Class 1 and compact section criterion are given as

\[
\frac{C_f}{C_f} \leq 0.9 \sqrt{\frac{225}{C_f} (\text{Eurocode 3})}
\]

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![Fig. 3. Shear flow and shear force acting on the cross-section of the I-girder with corrugated steel webs due to bending: (a) shear flow distribution; (b) shear force distribution and location of shear center.](image)
where \( E \) is the stress. In this study, 380 MPa, respectively. All analyzed models satisfied Eq. (6).

Different buckling modes (local, global, and interactive shear buckling) were proposed by Moon et al. [9] to maximize the condition to assure flexural–torsional buckling, also to minimize the effect of initial imperfection.

### Table 1
Dimensions of analysis models.

<table>
<thead>
<tr>
<th>Corrugation profiles</th>
<th>( a ) (mm)</th>
<th>( b ) (mm)</th>
<th>( c ) (mm)</th>
<th>( \theta ) (°)</th>
<th>( l_0 ) (mm)</th>
<th>( h_w ) (mm)</th>
<th>( t_w ) (mm)</th>
<th>( b_f ) (mm)</th>
<th>( t_f ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.P.1</td>
<td>300</td>
<td>260</td>
<td>300.2</td>
<td>29.9</td>
<td>560</td>
<td>1500</td>
<td>15</td>
<td>500</td>
<td>50</td>
</tr>
<tr>
<td>C.P.2</td>
<td>300</td>
<td>240</td>
<td>300</td>
<td>36.8</td>
<td>540</td>
<td>1500</td>
<td>15</td>
<td>500</td>
<td>50</td>
</tr>
</tbody>
</table>

\[
\beta_y \leq 0.38 \sqrt{\frac{E}{f_y}} \quad \text{(AISC)}
\]  

where \( \gamma \) is the largest width of the compression flange from the web-flange joint to the free edge of the flange, and \( f_y \) is yield stress. In this study, \( E \) and \( f_y \) are taken as 210,000 MPa and 380 MPa, respectively. All analyzed models satisfied Eq. (6).

Corrugated steel webs can fail by shear buckling with three different buckling modes (local, global, and interactive shear buckling). Moon et al. [9] proposed the condition to assure flexural–torsional buckling, also to minimize the effect of initial imperfection.

### 3.2. Verification of finite element models

The convergence test was first performed to obtain the refined mesh. C.P.2 in Table 1 was used for the mesh convergence test where the number of corrugation periods \( n = 10 \), and the end moment ratio \( M_{1}/M_{2} = 1 \). Fig. 6(a) shows the analysis models which have various sizes of mesh. Fig. 6(b) shows the results of the convergence study, where the x axis and the y axis denote the number of elements per flat panel and the elastic flexural–torsional strength of the I-girder with corrugated steel webs, respectively. From the results, finite element analysis results converged on a specific value when the number of the elements per horizontal panel was larger than three. Based on this result, more than four elements per horizontal panel were used for all analysis models.

The finite element analysis model was verified by comparing the result of previous theoretical research [6]. The profiles of the verification model were the same as those in Table 1 and uniform bending \( (M_{1}/M_{2} = 1) \) was applied to the girder. Verification results are summarized in Table 2. The results of finite element analyses agreed well with those of theory. The maximum discrepancy between finite element analysis results and theoretical results was 5.41% for Model 3.
3.3. Elastic flexural–torsional buckling behavior

The elastic flexural–torsional buckling behavior of the I-girder with corrugated steel webs under linear moment gradient is discussed herein. Fig. 7 shows the variation in dimensionless buckling load $M/M_{cr}$ with dimensionless out-of-plane deflection $U^2/l$, where critical moment $M_{cr}$ was obtained from the Southwell plot [18] and $U^2$ is the out-of-plane deformation which was calculated at the mid-span of the upper flange. Southwell plot is a well-known technique for determining the elastic critical load experimentally. Mandal and Calladine [18] reported that this technique also provides good prediction for a structure undergoing flexural–torsional buckling. From Fig. 7, it was found that the dimensionless load-displacement relationships become non-linear with increasing the moment ratio $M_1/M_2$ when $n$ is equal to 10, while bifurcation buckling occurs regardless of the moment ratio when $n$ is equal to 10.5. This is because different transverse deflection modes of flanges are generated depending on the numbers of corrugation periods. The flanges are deformed with a single curvature and the maximum out-of-plane deformation occurs at the mid-span of the I-girder when $n$ has an even number of half corrugations [ex. $n=10$ in Fig. 7(a) and (c)]. This out-of-plane deformation is directly proportional to the magnitude of shear force and acts as an initial imperfection to the I-girder. Thus, the dimensionless load-displacement relationships become non-linear with increasing $M_1/M_2$ for the I-girder with corrugated steel webs having an even number of half corrugations. On the other hand, for the I-girder with corrugated steel webs having an odd number of half corrugations, the transverse deformed shape of the flange has double curvatures. The out-of-plane deformation at mid-span is equal to zero and the maximum deformation occurs at the quarter point of the I-girder. Thus, the effects of the transverse deformation of the flange on buckling behavior can be negligible and bifurcation buckling occurs (ex. $n=10.5$ in Fig. 7(b) and (d)).

Transverse deformations of flanges at the mid-span of the I-girder were calculated for all analysis models and the results are shown in Fig. 8. In Fig. 8, $U^2(0.85)$ represents the transverse deformation of the flange at the mid-span when $M$ is equal to $0.85M_{cr}$. $U^2(0.85)/l$ was increased when increasing the moment ratio $M_1/M_2$ up to $M_1/M_2=0.75$ for $n=10$. However, the variation of $U^2(0.85)/l$ was negligible when $n$ is equal to 10.5. For the I-girder with corrugated steel webs having an even number of half corrugations, the maximum value of $U^2(0.85)$ was $l/100$, while $U^2(0.85)$ was less than $l/1000$ for the I-girder with corrugated steel webs having an odd number of half corrugations as shown in Fig. 8. Thus, for the I-girder with corrugated steel webs under linear moment gradient, an odd number of half corrugations is effective to reduce transverse deformation of the flange prior to buckling, and it leads to bifurcation type buckling.

3.4. Moment gradient correction factor

For the I-girder with corrugated steel webs subjected to an unequal end moment, it is assumed that the effects of moment

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**Table 2**
Verification results of the finite element models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Corrugation profiles</th>
<th>$n$</th>
<th>$M_1/M_2$</th>
<th>FEM (kN m)</th>
<th>Theory (kN m)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 C.P.1</td>
<td>10</td>
<td>-1</td>
<td>15,656</td>
<td>15,061</td>
<td>3.80</td>
<td></td>
</tr>
<tr>
<td>2 C.P.1</td>
<td>10.5</td>
<td>-1</td>
<td>14,353</td>
<td>13,833</td>
<td>3.62</td>
<td></td>
</tr>
<tr>
<td>3 C.P.2</td>
<td>10</td>
<td>-1</td>
<td>16,976</td>
<td>16,058</td>
<td>5.41</td>
<td></td>
</tr>
<tr>
<td>4 C.P.2</td>
<td>10.5</td>
<td>-1</td>
<td>15,544</td>
<td>14,733</td>
<td>5.22</td>
<td></td>
</tr>
</tbody>
</table>
gradient on the critical moment can be accounted for by using a moment gradient correction factor $C_b$. This approach is similar to that of the I-girder with flat webs. Thus, using the buckling strength $M_{oc}$ of the I-girder with corrugated webs under the uniform moment and $C_b$ factor, the flexural–torsional buckling strength $M_{cr}$ of the I-girder with corrugated steel webs subjected to non-uniform bending can be expressed as

$$M_{cr} = C_bM_{oc} = C_b\frac{\pi}{T} \sqrt{EIy_cG_{co}J_c} \sqrt{1 + W^2}, \quad W = \frac{\pi}{T} \sqrt{\frac{E \beta_c}{G_{co}J_c}}$$ \quad (8)

For the I-girder with flat webs subjected to an unequal end moment with no other loads through the span of the I-girder, the moment gradient correction factor $C_b$ in Eq. (8) is given by

$$C_b = 1.75 + 1.05\frac{M_A}{M_B} + 0.3\left(\frac{M_A}{M_B}\right)^2 \leq 2.3$$ \quad (9)

where $M_A$ is the smaller end moment. The ratio $M_A/M_B$ is positive when the girder bends in a double curvature and is negative for single curvatures. Eq. (9) was first proposed by Savadori [19] and several design codes [17,20] adopt Eq. (9) to consider the moment gradient effect. Also, the AISC [17] and AASHTO [20] specifications use the following equation for the moment gradient correction factor

$$C_b = \frac{12.5M_{max}}{3M_A + 4M_B + 3M_C + 2.5M_{max}}$$ \quad (10)

where $M_A$, $M_B$, and $M_C$ are the absolute values of the bending moment of the quarter, mid-span, and three-quarter point of the
l-girder, respectively, and \( M_{\text{max}} \) is the absolute maximum bending moment acting on the l-girder.

In this study, the moment gradient correction factor \( C_b \) was calculated from the results of the finite element analysis. Then, the results were compared with Eqs. (9) and (10). The moment gradient correction factor \( C_b \) for the l-girder with corrugated steel webs under linear moment gradient can be calculated as

\[
C_b = \frac{M_{\text{cr}}(\text{from FEM})}{M_{\text{cr}}(\text{from FEM})} \quad (11)
\]

where \( M_{\text{cr}} \) is the flexural–torsional buckling strength of the l-girder with corrugated steel webs under uniform bending.

Fig. 9 shows the comparison of \( C_b \) with current design codes described in Eqs. (9) and (10). It is found that Eqs. (9) and (10) give conservative values of \( C_b \) when increasing the moment ratio \( M_1/M_2 \). For accurate estimation of \( C_b \) for the l-girder with corrugated steel webs, the result of Lim et al. [21] is adopted to calculate the \( C_b \). Lim et al. [21] suggested \( C_b \) for l-beams with flat webs under linear moment gradient by using the Bubnov-Galerkin method. The suggested formula for \( C_b \) is expressed as

\[
C_b = \frac{2}{v \sqrt{1-(M_1/M_2)^2} + R_1(1+(M_1/M_2))^2} \quad (12)
\]

for simply supported l-beams in bending and torsion, where \( R_1 \) is the coefficient that depends on the linear moment gradient. From Eq. (12) and the results of finite element analyses, \( R_1 \) was obtained for the l-girder with corrugated steel webs under linear moment gradient by using least square approximation [22], and \( R_1 \) was equal to 0.1442. Fig. 10 shows a comparison of \( C_b \) obtained from finite element analysis with those from Eq. (12) with \( R_1 = 0.1442 \). It can be seen that the proposed \( C_b \) (Eq. (12) with \( R_1 = 0.1442 \)) provides good prediction of buckling strength for the l-girder with corrugated steel webs under linear moment gradient. The maximum error is 6.7%.

4. Inelastic flexural–torsional buckling strength of l-girder with corrugated steel webs under linear moment gradient

4.1. Theoretical inelastic flexural–torsional buckling strength

Once elastic flexural–torsional buckling strength is obtained, the inelastic flexural–torsional strength can be evaluated by using the buckling curve. In this study, the buckling curve from Eurocode 3 [16] was adopted. The flexural–torsional buckling strength in Eurocode 3 [16] is given by

\[
M_{\text{in}} = \lambda_{LT} M_p \quad (13)
\]

where \( M_p \) is the inelastic flexural–torsional strength of the l-girder with corrugated steel webs, \( M_p \) is the plastic moment of the section, and \( \lambda_{LT} \) is the reduction factor. According to Eurocode 3 [16], \( \lambda_{LT} \) is expressed as

\[
\lambda_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \phi_{LT}^2}} \leq 1 \quad (14a)
\]

\[
\phi_{LT} = 0.5[1 + \alpha_{LT}(\alpha_{LT} - 0.2) + \lambda_{LT}^2] \quad , \quad \lambda_{LT} = \sqrt{\frac{M_p}{M_{\text{cr}}}} \quad (14b)
\]

In Eq. (14), \( \lambda_{LT} \) is the buckling parameter, and \( \alpha_{LT} \) is the initial imperfection factor. \( \alpha_{LT} \) is taken as 0.34 for flexural–torsional buckling. To determine the buckling parameter \( \lambda_{LT} \), Eqs. (8) and (12) were used to calculate the elastic flexural–torsional buckling of the l-girder with corrugated steel webs under linear moment gradient.

4.2. Analysis models, initial imperfections, and residual stresses

For the non-linear inelastic analysis of the l-girder with corrugated steel webs under linear moment gradient, corrugation profiles 1 (C.P.1) and 2 (C.P.2) in Table 1 were used again. For each corrugation profile, the lengths of the l-girders \( l \) were selected in the range of 8400–22,960 mm so that the buckling parameter \( \lambda_{LT} \) varies from 0.675 to 1.37. Three different moment ratios \( (M_1/M_2 = 0.75, 0, \text{and } 0.75) \) were considered to investigate the effect of linear moment gradient.

Fig. 11 shows the stress–strain relationship of the steel used in this study where the modulus of elasticity \( E \) is 210,000 MPa, Poisson ratio \( v \) is 0.3, the yield stress \( f_y \) is 380 MPa, and the ultimate stress is 500 MPa. These properties are adopted based on the properties of HSB500, which is produced in South Korea as a structural steel.

The effects of the initial imperfection and residual stress on the flexural–torsional buckling are important for practical design purposes and should be considered in the analysis. In this study, to consider the initial imperfection, the 1st buckling shape obtained...
from Eigenvalue analysis was introduced to the model. The magnitude of initial imperfection was $l/1000$. Fig. 12 shows the shape and magnitude of initial imperfection. The shape of initial imperfection and the transverse flange bending shape prior to the buckling are similar when the number of corrugation periods is equal to 12 (even numbers of half corrugation) and the direction coincides when the sign of the initial imperfection is negative (refer direction of $x$ axis).

To investigate the effect of the direction of initial imperfection on the flexural–torsional buckling strength, both positive ($+$) and negative ($-$) directions were considered in the analysis. Fig. 13 shows the distribution of residual stress where maximum tensile and compressive residual stresses were equal to $f_y$ and $0.25f_y$, respectively [23]. The distribution of residual stress is varied along the longitudinal axis due to the eccentricity of the webs as shown in Fig. 13.

4.3. Comparison of theoretical buckling strength with finite element analysis

Fig. 14(a) and (b) show the analysis results without the effects of initial imperfection and residual stresses for I-girder with corrugated steel webs having an even number of half corrugations and an odd number of half corrugations, respectively. In Fig. 14, $x$ axis and the $y$ axis denote the buckling parameter $\lambda_{LT}$ and dimensionless buckling load $M_{cr}^{(0)}/M_0$, respectively. The analysis results agreed well with the elastic buckling curve and plastic limit regardless of corrugation profiles (C.P.1 and 2), the end moment ratio ($M_1/M_2 = -0.75$, 0, and 0.75), and number of corrugation periods $n$ as shown in Fig. 14.

Fig. 15(a) and (b) show the effects of the direction of initial imperfection on the flexural–torsional buckling strength of the
was lower than that of the I-girder having positive (+) initial imperfection. Furthermore, the flexural–torsional buckling strength of the I-girder having an even number of half corrugations was lower than that of the I-girder having an odd number of half corrugations. This is because the assumed initial imperfection and transverse flange bending have the same direction and single curvature when the sign of the initial imperfection is negative and the I-girder has an even number of half corrugations. The average strength reductions for the I-girder with corrugated steel webs having even and odd numbers of half corrugations were 6.7 and 2.7%, respectively.

Fig. 16 shows the comparison of analysis results including the effects of initial imperfection and residual stress with buckling curve. The flexural–torsional buckling strength was considerably reduced by the effects of initial imperfection and residual stresses. The strength reduction was largest when the end moment ratio $M_1/M_2$ was equal to $-0.75$ as shown in Fig. 16. For all analyzed models, the buckling curve from Eurocode 3 [16] conservatively predicted the flexural–torsional buckling strength of the I-girder with corrugated steel webs. Thus, the buckling curve from Eurocode 3 [16] with the proposed elastic flexural–torsional buckling strength of the I-girder (Eqs. (8) and (12)) can provide a reasonably conservative flexural–torsional buckling strength prediction of the I-girder with corrugated steel webs under linear moment gradient.

5. Summary and conclusions

This paper investigated the flexural–torsional buckling of the I-girder with corrugated steel webs under linear moment gradient. A series of finite element analyses was conducted and the effects of the number of corrugation periods, corrugation profiles, and end moment ratios were examined. From the results, it was found that the flange of the girder deformed in the out-of-plane direction with a single curvature and it acts as an initial imperfection when corrugated steel webs have an even number of half corrugations. This results in a non-linear load-displacement relationship of the I-girder. On the other hand, bifurcation buckling occurs when the I-girder has an odd number of half corrugations. Thus, an odd number of half corrugations is effective to reduce transverse deformation of the flange prior to buckling, and leads to bifurcation type buckling.

The moment gradient correction factor $C_b$ for the I-girder with corrugated steel webs under linear moment gradient was proposed as Eq. (12). The comparative study shows that the current design codes give conservative values of $C_b$ in high end moment ratio $M_1/M_2$, while the proposed equation for $C_b$ agrees well with the results of the finite element analyses. Finally, the inelastic flexural–torsional buckling strength considering the effects of material inelasticity, initial imperfection, and residual stress was investigated. The inelastic buckling strength varied depending on the numbers of periods of the corrugations and direction of initial imperfection. However, for all analyzed models, the buckling curve from Eurocode 3 with the proposed moment gradient correction factor $C_b$ gave a reasonably conservative prediction of the I-girder with corrugated steel webs under linear moment gradient.

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